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Endogenous timing with price competition when a public firm supplies a private rival

Temporización endógena con competencia en precios cuando una empresa pública provee a un rival privado

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ABSTRACT

We study the endogenous order of firms' moves in a price-setting mixed duopoly where a public firm not only competes with a private firm in the retail market but also supplies an input to such rival. Markets where the traditional separation between firms selling to ultimate consumers and their suppliers is not observed, as in the framework studied here, are common in practice. We focus on the usual case where the input price is regulated and find that the traditional result of simultaneous price setting may not hold, and sequential price setting is instead likely to emerge in a wide variety of circumstances.

RESUMEN

Estudiamos el orden endógeno de los movimientos de las empresas en un duopolio mixto con competencia en precios cuando una empresa pública no solamente compite en el mercado final con una empresa privada, sino que además la provee de un insumo intermedio. Los mercados donde no se observa la separación tradicional entre las empresas que venden a los consumidores finales y sus proveedores, como en el marco que se estudia en este artículo, son comunes en la práctica. Nos concentramos en el caso usual en que el precio del insumo es regulado y encontramos que el resultado tradicional de fijación simultánea de precios puede no presentarse y en cambio es probable que ocurra la fijación de precios secuencial en una gran variedad de circunstancias.

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INTRODUCTION

Markets where private firms compete with government-owned firms are often observed in major economic sectors such as energy and telecommunications. The study of these markets, subject to different strategic interactions than pure private markets, has been the object of a large literature in recent decades. One issue that has received much attention is the endogenous order of moves of the public and private firms that participate in them. As Pal (1998) stresses, different order of firms' moves often lead to substantially different results, and rather than assuming exogenously this order, it should result from firms' decisions. To study these decisions, Pal (1998) considers the observable delay game by Hamilton and Slutsky (1990) applied to a quantity-setting mixed oligopoly, in which firms first decide when they will choose their quantities and then decide at the previously chosen time the amount they will produce.

Bárcena-Ruiz (2007) also applies the Hamilton and Slutsky (1990) framework to study endogenous timing. But instead of considering the quantity-setting case he analyzes a differentiated mixed duopoly under price competition. He finds that firms set prices simultaneously. Other analysis under price competition are Bárcena-Ruiz and Sedano (2011), Naya (2015), Din and Sun (2016) and Lee and Xu (2018).

The previous papers assume that the traditional separation between firms selling to ultimate consumers and their suppliers holds. Yet, as Arya and Mittendorf (2018) point out, there are markets where this separation is not observed, as in e-commerce, where a manufacturer uses its own on-line channel and continues to sell to independent retailers, and in markets where a substantial capacity is needed for input production –as in utilities– and a dominant Vertically Integrated Producer (VIP) exists. In these markets a firm that sells to end consumers is also a supplier of a retail competitor.

The telecommunication industry is one among other industries where there are often public firms that compete with private firms and also provide key inputs to them. For example, government-owned Antel in Uruguay, in addition to offering mobile phone services competing with private mobile phone companies, is the only provider of fixed phone services. Thus, Antel provides a key input in the provision of its rivals' mobile to fixed calls.

In contrast with previous literature, we analyze endogenous timing under price competition in mixed markets where the separation between firms selling to end consumers and their suppliers does not hold. We examine a differentiated mixed duopoly where a public firm not only sells goods in the retail market but also manufactures an input needed to produce this good and sells it to its retail private competitor. We focus on the usual situation of regulated input prices and consider both an exogenous and an endogenous regulator-set input price.

Our setting is similar to Arya and Mittendorf (2018) with the difference that they study a private duopoly, and we consider a mixed duopoly. It is also akin to Fernández-Ruiz (2024), who does consider a mixed duopoly, but studies the quantity competition case instead of the price competition case.

We find that under an exogenous input price the traditional result of simultaneous price setting may not hold, and sequential price setting is instead likely to emerge, while in the endogenous case a regulator that maximizes consumer surplus subject to the public firm making non-negative profits induces sequential price setting.

The rest of the paper is organized as follows. Section I introduces the model. Section II analyzes the case of an exogenous input price. Section III considers the case of an endogenous, regulator-set input price. A final section offers some concluding thoughts.

I. THE MODEL

Following Singh and Vives (1984) we consider an economy with a competitive numeraire sector and a monopolistic sector where each of two firms produces a differentiated good while, on the demand side, a continuum of consumers have utility functions separable and linear in the numeraire good and, thus, income effects on the monopolistic sector are absent and partial equilibrium analysis can be performed. In the monopolistic sector there is a public firm (firm 0) that maximizes social welfare[1] and a private firm (firm 1) that maximizes its own profits.

1. The assumption that the public firm maximizes social welfare is standard in the mixed oligopoly literature. A different approach can be found in, for example, Zhang and Zhong (2015).

The utility function of the representative consumer is[2]

$$U(q_0, q_1) = a(q_0 + q_1) - \frac{1}{2}(q_0^2 + 2\gamma q_0 q_1 + q_1^2) \tag{1}$$

where q_i and p_i, $i = 0,1$, denote firm i's quantity and price, respectively, and $\gamma\epsilon(0,1)$ measures the degree of substitutability of the goods.[3] Maximization of $U(q_0, q_1) - p_0 q_0 - p_1 q_1$ leads to demand functions

$$q_i = \frac{a}{1+\gamma} - \frac{1}{1-\gamma^2}p_i + \frac{\gamma}{1-\gamma^2}p_j \qquad i,j = 0,1, i \neq j \tag{2}$$

Production of the final goods requires an input, whose price is regulated, that firm 0 manufactures both for use in its own production process and for sale to its retail competitor. Thus, firm 0 is both firm 1's competitor and firm 1's supplier. Each unit of input yields one unit of the final good,[4] and the costs of producing the input are quadratic:[5] $C(q_0, q_1) = \frac{1}{2}(q_0 + q_1)^2$.

Firm 0´s profits are:

$$\Pi_0 = p_0 q_0 + w q_1 - C(q_0, q_1) \tag{3}$$

where w denotes the input price, with $\frac{a(\gamma+1)(\gamma^2+2\gamma-2)}{\gamma^2+2\gamma+2} = w_{min} \leq w \leq w_{max} = \frac{-a(\gamma^2-\gamma-4)}{3\gamma+4}$ to ensure non-negative outputs.

The first term in the RHS in (3) represents the revenues that firm 0 obtains from selling the final good in the retail market, the second term represents the revenues it obtains from selling the input to firm 1, and the last term represents the costs of producing the input.

Firm 1 obtains revenues only in the retail market and its profits are:

$$\Pi_1 = (p_1 - w) q_1 \tag{4}$$

By replacing the demand functions in (2) into consumer surplus $CS = U(q_0, q_1) - p_0 q_0 - p_1 q_1 = a(q_0 + q_1) - \frac{1}{2}(q_0^2 + 2\gamma q_0 q_1 + q_1^2) - p_0 q_0 - p_1 q_1$ we can write it as:

$$CS = \frac{2(1-\gamma)(a^2 - a(p_0 + p_1)) + p_0^2 + p_1^2 - 2\gamma p_0 p_1}{2(1-\gamma^2)} \tag{5}$$

Social welfare is the sum of producer surplus ($PS = \Pi_0 + \Pi_1$) and consumer surplus:

$$SW = PS + CS = \Pi_0 + \Pi_1 + CS \tag{6}$$

2. This is a simplified version of Singh and Vives (1984).
3. This framework is commonly used in the mixed oligopoly literature. See for example Barcena-Ruiz (2007), who studies endogenous timing in a mixed duopoly under price competition keeping the traditional assumption of separation between suppliers and retailers. See also Fernández-Ruiz (2024), who departs from this assumption as we do here, but considers quantity competition instead of price competition.
4. Notice that both firm 0 and firm 1 can transform one unit of input into one unit of output, but consumers are not capable of doing this. This precludes the possibility of consumers buying the input for its direct consumption.
5. Note that total input production equals $(q_0 + q_1)$ because the input is required for producing both final goods. The quadratic input cost function assumed here implies an increasing marginal cost. Models with upstream firms in vertical relationships with increasing marginal costs include Matsushima (2017) and King (2013). These costs appear when there are capacity or financial constraints as firms approach these constraints.

II. EXOGENOUS INPUT PRICE

Since each firm can set its retail price in period 1 or in period 2, there are three possible order or moves: i) simultaneous choices, if both firms set their prices in period 1 or if they both do it in period 2, ii) public leadership, if firm 0 sets its price in period 1 and firm 1 sets its price in period 2, and iii) private leadership, if firm 1 sets its price in period 1 and firm 0 sets its price in period 2.

Consider first simultaneous price setting.

Firm 0 chooses its price to maximize SW, leading to its reaction function

$$p_0 = \frac{p_1(\gamma^2 + 2\gamma - 1) + 2a(1 - \gamma)}{2} \tag{7}$$

It is worth noting the absence of the input price in the public firm's reaction function. To gain intuition, notice that the input price is absent both in the expression for consumer surplus, as given in (5), and in the expression for producer surplus $PS = \Pi_0 + \Pi_1 = p_0 q_0 + p_1 q_1 - \frac{1}{2}(q_0 + q_1)^2$, the two components of SW.

In contrast with the traditional upward sloping firms' reaction functions that arise when suppliers and retailers are separated,[6] here firm 0's reaction function is upward sloping only for $\gamma > 0.4142$. When $\gamma < 0.4142$ firm 0's price decreases in p_1. To gain intuition, consider the polar case where γ approaches zero, so that there is no relationship between the goods on the demand side: final goods are independent goods. It is then optimal for the public firm to set its price p_0 equal to marginal cost, and thus p_0 would not depend on the private firm's price p_1 under the traditional separation between suppliers and retailers. But here an increase in p_1 translates into a reduction in the marginal cost of producing the input because it reduces firm 1's production and thus induces firm 0 to set a lower price p_0. Thus, even when final goods are independent, if the public firm supplies an input to its private rival there is still a link between the two firms' price choices. As γ increases, the demand-side link becomes more important and for a high enough value of γ the traditional upward-sloping reaction function is obtained.

Firm 1 chooses its price to maximize Π_1, leading to its reaction function

$$p_1 = \frac{(p_0 - a)\gamma + w + a}{2} \tag{8}$$

Firm 1's reaction function is always upward sloping.

Solving (7) and (8) yields:[7]

$$p_0^s = \frac{a\gamma^3 + (a - w)\gamma^2 + (a - 2w)\gamma + w - 3a}{\gamma^3 + 2\gamma^2 - \gamma - 4} \tag{9}$$

$$p_1^s = \frac{2a\gamma^2 - 2w - 2a}{\gamma^3 + 2\gamma^2 - \gamma - 4} \tag{10}$$

6. i.e. prices are strategic complements.

7 Superscripts S, 0L and 1L denote simultaneous price-setting, public leadership and private leadership.

$$\Pi_1^s = \frac{(1-\gamma^2)(w\gamma + 2w - 2a)^2}{(\gamma^3 + 2\gamma^2 - \gamma - 4)^2} \tag{11}$$

$$SW^s = \frac{(a\gamma^2 - w - a)(a\gamma^3 + a\gamma^2 + w\gamma - a\gamma + 3w - 5a)}{(\gamma^3 + 2\gamma^2 - \gamma - 4)^2} \tag{12}$$

Consider now firm 1's leadership. Firm 0 chooses its price in period 2 according to (7). Firm 1 chooses its price in period 1 to maximize its own profits, anticipating firm 0's response, which yields

$$P_1^{1L} = \frac{w\gamma + 2w + 2a}{2\gamma + 4} \tag{13}$$

$$p_0^{1L} = \frac{w\gamma^3 + 4w\gamma^2 - 2a\gamma^2 + 3w\gamma - 2w + 6a}{4(\gamma + 2)} \tag{14}$$

$$\Pi_1^{1L} = \frac{(w\gamma + 2w - 2a)^2}{8(\gamma + 2)} \tag{15}$$

$$SW^{1L} = -\frac{(w\gamma + 2w + 2a)(w\gamma^2 + 5w\gamma - 6a\gamma + 6w - 10a)}{16(\gamma + 2)^2} \tag{16}$$

Consider finally firm 0's leadership. Firm 1 chooses its price in period 2 according to (8). Firm 0 chooses its price in period 1 to maximize social welfare, anticipating firm 1's response, which yields

$$p_0^{0L} = \frac{a\gamma^3 - w\gamma^2 + 2a\gamma^2 - w\gamma + w - 3a}{2\gamma^3 + 3\gamma^2 - 2\gamma - 4} \tag{17}$$

$$p_1^{0L} = -\frac{a\gamma^4 - w\gamma^3 - a\gamma^3 - 2w\gamma^2 - 5a\gamma^2 + w\gamma + a\gamma + 4w + 4a}{2(2\gamma^3 + 3\gamma^2 - 2\gamma - 4)} \tag{18}$$

$$\Pi_1^{0L} = \frac{(1 - \gamma^2)(a\gamma^2 + 3w\gamma - a\gamma + 4w - 4a)^2}{4(2\gamma^3 + 3\gamma^2 - 2\gamma - 4)^2} \tag{19}$$

$$SW^{0L} = \frac{a^2\gamma^3 - 2aw\gamma^2 + 5a^2\gamma^2 + w^2\gamma - a^2\gamma + 3w^2 - 2aw - 5a^2}{4(2\gamma^3 + 3\gamma^2 - 2\gamma - 4)} \tag{20}$$

Comparison of firms' prices yields:

Proposition 1

i) $p_0^{1L} > p_0^S$, while $p_1^{1L} < (>)\, p_1^S$ if $\gamma < (>)\, 0.4142$

ii) $p_0^{0L} > (<)\, p_0^S$ and $p_1^{0L} > (<)\, p_1^S$ if $w < (>)\, w_r$

with $w_r = \frac{a(\gamma^2+1)}{\gamma+3}$, $w_{min} < w_r < w_{max}$

Proposition 1 tells us that the private firm uses its leadership role to induce the public firm to set a higher price than under simultaneous price setting ($p_0^{1L} > p_0^S$). To induce this higher price, the private firm acts differently depending on the degree of substitutability of the goods: when $\gamma < 0.4142$, it reduces its price relative to the simultaneous case ($p_1^{1L} < p_1^S$) because the public firm's reaction function is downward sloping, and when $\gamma < 0.4142$ it increases its price relative to the simultaneous case ($p_1^{1L} > p_1^S$) because the public firm's reaction function is upward sloping.

The comparison between prices under public leadership and under simultaneous choices (p_0^{0L} versus p_0^S and p_1^{0L} versus p_1^S) depends on the level on the input price w: both firms' prices are higher under public leadership than with simultaneous choices if $w < w_r$ and both are lower if $w > w_r$. To gain intuition, notice from the private firm's reaction function that the private firm's price is increasing both in the public firm's price and in the input price. Notice also that, given a fixed level of production, a change in the input price does not directly affect social welfare (the public firm's objective function) because it only redistributes profits between the public firm and the private firm. If the input price is high ($w > w_r$) the public firm uses its leadership role to induce a reduction in the otherwise high private firm's price resulting from such an elevated input price. It induces this reduction by setting a lower price than under simultaneous choices. If the input price is low ($w < w_r$) the public firm raises its price relative to simultaneous price setting and induces the private firm to increase its price above the otherwise low level that results from the low input price.

Proposition 2

i) $\Pi_1^{0L} > (<) \Pi_1^S$ if $w < (>) w_r$ while $\Pi_1^{1L} > \Pi_1^S$

ii) If $w > w_v$, $SW^{1L} > (<) SW^S$ when $\gamma < (>) 0.4142$, and if $w < w_v$, $SW^{1L} > (<) SW^S$ when $\gamma > (<) 0.4142$, while $SW^{0L} > SW^S$

with $w_v = \frac{2a(\gamma^4+\gamma^3-3\gamma^2-7\gamma-8)}{(\gamma+2)(\gamma+3)(\gamma^3+2\gamma^2-\gamma-8)}$, $w_{min} < w_v < w_{max}$, $w_r < (>) w_v$ if $\gamma < (>) 0.4142$

When the input price is lower than w_r the private firm's profits are higher under public leadership than under simultaneous choices because the public firm sets a higher price when it is the leader than in the simultaneous case. The opposite happens when the input price is higher than w_r.

Comparison of social welfare between private leadership and simultaneous choices depends on both the level of the input price and the degree of substitutability of the products. Remember that a high input price increases the private firm's price both under private leadership and under simultaneous choices, but it does not directly affect social welfare. Proposition 2 tells us that if the input price is higher than w_v, the timing with the smaller private firm's price, as given in proposition 1, yields a higher social welfare reflecting the fact that it reduces the otherwise high private firm's price resulting from this input price. On the contrary, when the input price is lower than w_v, the timing with the higher private firm's price yields the higher social welfare because it increases the otherwise low private firm's price.

We can now analyze firms' timing choices using proposition 2 to obtain:

Proposition 3

i) Assume $\gamma < 0.4142$. Then $w_r < w_v$ and in equilibrium there is public leadership if $w < w_r$, simultaneous price setting if $w_r < w < w_v$ and private leadership if $w_v < w$.

ii) Assume $\gamma > 0.4142$. Then $w_v < w_r$ and there are two equilibria, one with public leadership and the other one with private leadership, if $w < w_v$, while in equilibrium there is public leadership if $w_v < w < w_r$ and simultaneous price setting if $w_r < w$.

The traditional result that firms choose prices simultaneously only holds for intermediate input prices when the degree of substitutability of the products is low and for high input prices otherwise. In all the other cases firms choose prices sequentially.

To gain some insight, let us focus on the case of low input prices keeping in mind that the traditional result of early simultaneous price-setting holds when both firms prefer choosing prices simultaneously over being a follower. Here, in contrast, this condition does not hold because when the input price is low the private firm prefers being a follower over simultaneous price setting. If $\gamma < 0.4142$, public leadership emerges as the only equilibrium since the public firm does dislike being a follower. When $\gamma > 0.4142$, private leadership appears as a second equilibrium –in addition to public leadership– because the public firm also prefers being a follower over simultaneous price setting.

IV. THE REGULATOR'S CHOICE

We now consider a regulator interested in maximizing consumer surplus[8] (as in Chen and Sappington (2009) and Wolak (1994)) under the constraint that the public firm obtains non-negative profits. The regulator can induce any equilibrium in Proposition 3 if it sets the appropriate input price. We consider the following two-step procedure similar to that in Arya and Mittendorf (2018):[9] first, we find the input price that maximizes consumer surplus among the prices that induce i) public leadership, ii) private leadership, and iii) simultaneous price-setting. Second, we find the regulator's preferred choice among these three options. Formally, let

$$w_{OL} = \underset{w}{\operatorname{argmax}} \{ CS^{0L} \ s.t. \ \Pi_0^{0L} \geq 0 \text{ and } w_{min} \leq w \leq w_r \} \tag{21}$$

$$w_S = \underset{w}{\operatorname{argmax}} \{ CS^{S} \ s.t. \ \Pi_0^{S} \geq 0 \text{ and } \begin{Bmatrix} w_r \leq w \leq w_v & \text{if } \gamma < 0.4142 \\ w_r \leq w \leq w_{max} & \text{if } \gamma > 0.4142 \end{Bmatrix} \tag{22}$$

$$w_{1L} = \underset{w}{\operatorname{argmax}} \{ CS^{1L} \ s.t. \ \Pi_0^{1L} \geq 0 \text{ and } \begin{Bmatrix} w_v \leq w \leq w_{max} & \text{if } \gamma < 0.4142 \\ w_{min} \leq w \leq w_v & \text{if } \gamma > 0.4142 \end{Bmatrix} \tag{23}$$

Then we have:

Lemma 1

i) $w_{OL} = \begin{cases} w_{OLC} & \text{if } \gamma < 0.9385 \\ w_{min} & \text{if } \gamma > 0.9385 \end{cases}$ where w_{OLC} satisfies $\Pi_0^{0L}|_{w=w_{OLC}} = 0$

ii) $w_S = w_r$

iii) $\begin{cases} w_{1L} \epsilon \{w_v, w_{max}\} & \text{if } \gamma < 0.4142 \\ w_{1L} \epsilon \{w_{min}, w_{1LC}\} & \text{if } \gamma > 0.4142 \end{cases}$ where w_{1LC} satisfies $\Pi_0^{1L}|_{w=w_{1LC}} = 0$

and the choice among these options leads to:

Proposition 4

The regulator induces public leadership if $\gamma < 0.4142$ or $\gamma > 0.6586$ and it induces private leadership if $0.4142 < \gamma < 0.6586$.

To gain some insight on this result, let us first focus on the fact that the regulator does not induce simultaneous price setting. The regulator prefers to induce public leadership by setting the input price that

8. See Fernández-Ruiz (2024) for an example that motivates this assumption. We show in the appendix that under the alternative assumption that the regulator maximizes social welfare its optimal choice is not unique and both sequential and simultaneous price setting can be optimally induced.

9. With the difference that here we maximize the regulator's utility, CS, instead of the VIP's utility.

allows the public firm to just break even, w_{OLC}, instead of inducing simultaneous price setting by raising the input price to $w = w_r$. We can decompose this change in two steps. First, keeping public leadership, this increase damages consumers. Second, given $w = w_r$, simultaneous price setting replaces public leadership. But this change in timing benefits consumers only for low input prices, because only in such a case (see proposition 1) does public leadership lead to higher prices than simultaneous price setting. Therefore, the regulator prefers public leadership over simultaneous price setting.

Let us focus now on the choice between public and private leadership. When $\gamma < 0.4142$, the regulator can induce public leadership by setting the input price that allows the public firm to just break even, w_{OLC}. If it wants to induce private leadership, it must set a higher input price $w_{1L} \epsilon \, \{w_v, w_{max}\}$. This increase hurts consumers. Given the higher input price w_v, private leadership replaces public leadership. This change in timing causes an increase in the price of the public firm, adding another consumer-hurting effect, such that even a decrease in the price of the private firm is insufficient to reverse the overall impact. Thus, public leadership prevails. When $\gamma > 0.4142$, low input prices are also compatible with private leadership. In fact, when γ is very large the optimal way to induce both private leadership and public leadership is by setting $w = w_{min}$ and the regulator prefers again public leadership because then private leadership entails higher prices for both goods. In contrast, when γ takes on lower values in the range $\gamma > 0.4142$, the optimal way to induce both private leadership and public leadership is by setting input prices that allow the public firm to just break even. In this case, prices are lower under private leadership than under public leadership if the degree of substitutability is low and they are higher otherwise and thus there is a range of intermediate γ values for which private leadership is preferred.

CONCLUSION

We have extended the analysis of endogenous timing in a price-setting mixed duopoly to the case where a public firm sells an input to a downstream private rival under regulated input prices. We have found that the endogenous order of price setting differs substantially from the one observed under the commonly assumed separation between suppliers and retailers, since the traditional result of simultaneous price setting may not hold and sequential price setting is likely to emerge.

To relate our results with previous literature, let us notice that similar findings have been obtained under several environments that depart from the standard price-setting mixed duopoly in various ways. Such is the case in Bárcena-Ruiz and Sedano (2011) who also consider a mixed duopoly under price competition, but depart from the usual assumption that the public firm maximizes social welfare and assume instead that this firm's objective function assigns different weights to consumer surplus and producer surplus. It happens in Naya (2015) as well, who examines the case of a mixed duopoly where instead of competing with a public firm, a private firm competes with a firm with mixed ownership, since this firm is partially privatized. And it is also the case in Lee and Xu (2018) who examine a price-setting mixed duopoly where production generates an environmental externality, there is an optimal tax on this externality, and firms can engage in activities that reduce such externality. In these three settings the traditional result of simultaneous price setting may not hold, and sequential price setting may instead appear, just as in the model analyzed here. Thus, although the results in this model are obtained under a particular set of assumptions that refer to a specific industry configuration, they are in line with previous literature that arrives at similar findings under environments that differ from the standard mixed duopoly model and, in this sense, reinforces the idea that simultaneous price setting may not prevail when we abandon the standard mixed duopoly assumptions.

It is worth noticing as well that our results parallel those in Fernández-Ruiz (2024) where it is also shown –employing a framework like the one in the current paper– that the traditional timing results are changed when a public firm supplies a private competitor, with the difference that quantity competition –instead of the price competition considered here– is assumed.

The model in the current paper can be extended along the lines developed in Bárcena-Ruiz and Sedano (2011), Naya (2015) or Lee and Xu (2018) mentioned above, for instance, analyzing the case where instead of being a public firm, the firm that produces the input has mixed ownership, as in Naya (2015). Another possible extension is the study of the case where the private firm is foreign owned as in Matsumura (2003) or Bárcena-Ruiz and Sedano (2011).[10] The analysis of this extension is worth undertaking because the introduction of foreign firms affects the outcomes and they in fact often appear in mixed oligopolies.

APPENDIX

Proof of Proposition 1

i.i) $p_0^{1L}-p_0^S=\frac{\gamma(w\gamma+2w-2a)(\gamma^2+2\gamma-1)^2}{4(\gamma+2)(\gamma^3+2\gamma^2-\gamma-4)}$ is positive because it is decreasing in w and it is positive even when $w=w_{max}$ (then $p_0^{1L}-p_0^S=-\frac{a\gamma^3(\gamma+1)(\gamma^2+2\gamma-1)^2}{4(\gamma+2)(3\gamma+4)(\gamma^3+2\gamma^2-\gamma-4)}>0$)

i.ii) $p_1^{1L}-p_1^S=\frac{\gamma(w\gamma+2w-2a)(\gamma^2+2\gamma-1)}{2(\gamma+2)(\gamma^3+2\gamma^2-\gamma-4)}$ has the same sign as $(\gamma^2+2\gamma-1)$ because $(w\gamma+2w-2a)$ is negative (it is increasing in w and it is equal to $-\frac{a\gamma^2(\gamma+1)}{3\gamma+4}<0$ when $w=w_{max}$) and $\gamma^3+2\gamma^2-\gamma-4$ is also negative. Now, $(\gamma^2+2\gamma-1)>(<)\,0$ when $\gamma>(<)\,0.4142$.

ii.i) $p_0^{0L}-p_0^S=\frac{(1-\gamma)\gamma(\gamma+1)^2(a\gamma^2-w\gamma-3w+a)}{(\gamma^3+2\gamma^2-\gamma-4)(2\gamma^3+3\gamma^2-2\gamma-4)}$ is decreasing in w, it is positive when $w=w_{min}$ (then $p_0^{0L}-p_0^S=\frac{-2a(1-\gamma)\gamma(\gamma+1)^2}{(\gamma^2+2\gamma+2)(\gamma^3+2\gamma^2-\gamma-4)}>0$), negative when $w=w_{max}$ (then $p_0^{0L}-p_0^S=\frac{2a(1-\gamma)\gamma(\gamma+1)^2}{(3\gamma+4)(\gamma^3+2\gamma^2-\gamma-4)}<0$) and it is equal to zero when $w=w_r$.

ii.ii) $p_1^{0L}-p_1^S=\frac{(1-\gamma)\gamma^2(\gamma+1)^2(a\gamma^2-w\gamma-3w+a)}{2(\gamma^3+2\gamma^2-\gamma-4)(2\gamma^3+3\gamma^2-2\gamma-4)}$ is decreasing in w, it is positive when $w=w_{min}$ (then $p_1^{0L}-p_1^S=\frac{-a(1-\gamma)\gamma^2(\gamma+1)^2}{(\gamma^2+2\gamma+2)(\gamma^3+2\gamma^2-\gamma-4)}>0$), negative when $w=w_{max}$ (then $p_1^{0L}-p_1^S=\frac{a(1-\gamma)\gamma^2(\gamma+1)^2}{(3\gamma+4)(\gamma^3+2\gamma^2-\gamma-4)}<0$) and it is equal to zero when $w=w_r$.

Proof of Proposition 2

i.i) $\Pi_1^{0L}-\Pi_1^S=\frac{(1-\gamma)\gamma^2(\gamma+1)^2(a\gamma^2-w\gamma-3w+a)G(w)}{4(\gamma^3+2\gamma^2-\gamma-4)^2(2\gamma^3+3\gamma^2-2\gamma-4)^2}$

with $G(w)=w(7\gamma^4+24\gamma^3+13\gamma^2-32\gamma-32)+a(\gamma^5+\gamma^4-15\gamma^3-23\gamma^2+16\gamma+32)$

The result holds because a) $(a\gamma^2-w\gamma-3w+a)>(<)\,0$ if $w<(>)\,w_r$ (since it is decreasing in w and vanishes when $w=w_r$) and b) $G(w)>0$ since it is decreasing in w and it is positive even when $w=w_{max}$ ($G(w_{max})=-\frac{2a\gamma^2(\gamma+1)(2\gamma^3+3\gamma^2-2\gamma-4)}{3\gamma+4}>0$)

i.ii) $\Pi_1^{1L}-\Pi_1^S=\frac{\gamma^2(w\gamma+2w-2a)^2(\gamma^2+2\gamma-1)^2}{8(\gamma+2)(\gamma^3+2\gamma^2-\gamma-4)^2}>0$)

10. Other mixed oligopoly models with foreign firms and endogenous timing include Lu (2006), Heywood and Ye (2009) and Kawasaki, Ohkawa and Okamura (2020).

ii.i) $SW^{1L} - SW^{S} = \frac{\gamma(-w\gamma - 2w + 2a)(\gamma^2 + 2\gamma - 1)F(w)}{16(\gamma+2)^2(\gamma^3 + 2\gamma^2 - \gamma - 4)^2}$

with $F(w) = w(\gamma+2)(\gamma+3)(\gamma^3 + 2\gamma^2 - \gamma - 8) – 2a(\gamma^4 + \gamma^3 - 3\gamma^2 - 7\gamma - 8)$.

The result holds because a) $(-w\gamma - 2w + 2a) > 0$, since it is decreasing in w and it is positive even when $w = w_{max}$ (it is then equal to $\frac{a\gamma^2(\gamma+1)}{3\gamma+4} > 0$), b) $(\gamma^2 + 2\gamma - 1) > (<) 0$ if $\gamma > (<) 0.4142$, and c) $F(w) > (<) 0$ if $w < (>)$ w_v.

ii.ii) $SW^{0L} - SW^{S} = \frac{-(1-\gamma)\gamma^2(\gamma+1)^2(a\gamma^2 - w\gamma - 3w + a)^2}{4(\gamma^3 + 2\gamma^2 - \gamma - 4)^2(2\gamma^3 + 3\gamma^2 - 2\gamma - 4)} > 0$).

Proof of Proposition 3

0) $w_r - w_v = \frac{a\gamma(\gamma^2 + 2\gamma - 1)(\gamma^3 + 2\gamma^2 - \gamma - 4)}{(\gamma+2)(\gamma+3)(\gamma^3 + 2\gamma^2 - \gamma - 8)} < (>) 0$ if $\gamma < (>) 0.4142$ because it has the same sign as $(\gamma^2 + 2\gamma - 1)$.

i) Assume $\gamma < 0.4142$. If $w < w_r < w_v$ it is a dominant strategy for firm 0 to set its price in period 1 ($SW^{0L} > SW^{S} > SW^{1L}$), while firm 1 prefers being the follower to simultaneous choices ($\Pi_1^{0L} > \Pi_1^{S}$): there is public leadership. If $w_r < w < w_v$ it is a dominant strategy for both firms to set their prices in period 1 ($SW^{0L} > SW^{S} > SW^{1L}$ and $\Pi_1^{1L} > \Pi_1^{S} > \Pi_1^{0L}$): there is simultaneous price setting. If $w_r < w_v < w$ it is a dominant strategy for firm 1 to set is price in period 1 ($\Pi_1^{1L} > \Pi_1^{S} > \Pi_1^{0L}$) while firm 0 prefers being the follower to simultaneous choices ($SW^{1L} > SW^{S}$): there is private leadership.

ii) Assume $\gamma > 0.4142$. If $w < w_v < w_r$ each firm prefers being the leader or being the follower to simultaneous choices ($SW^{0L} > SW^{S}$, $SW^{1L} > SW^{S}$, $\Pi_1^{1L} > \Pi_1^{S}$, $\Pi_1^{0L} > \Pi_1^{S}$). Thus, there are two equilibria: public leadership and private leadership. If $w_v < w < w_r$, it is a dominant strategy for firm 0 to set its price in period 1 ($SW^{0L} > SW^{S} > SW^{1L}$), while firm 1 prefers being the follower to simultaneous choices ($\Pi_1^{0L} > \Pi_1^{S}$): there is public leadership. If $w_v < w_r < w$ it is a dominant strategy for both firms to set their prices in period 1 ($SW^{0L} > SW^{S} > SW^{1L}$ and $\Pi_1^{1L} > \Pi_1^{S} > \Pi_1^{0L}$): there is simultaneous price setting.

Proof of Lemma 1

i) CS^{0L} is decreasing in w. $\Pi_0^{0L} < (>) 0$ when $w = w_{min}$ if $\gamma < (>) 0.9385$, $\Pi_0^{0L} > 0$ when $w = w_r$ and Π_0^{0L} is increasing in w.

ii) CS^{S} is decreasing in w and $\Pi_0^{S} > 0$ when $w = w_r$.

iii.i) CS^{1L} is convex in w and $\Pi_0^{1L} > 0$ both when $w = w_v$ and when $w = w_{max}$.

iii.ii) CS^{1L} is decreasing in w over the range $[w_{min}, w_v]$, and $\Pi_0^{1L} < (>) 0$ when $w = w_{min}$ if $\gamma < (>) 0.8584$, $\Pi_0^{1L} > 0$ when $w = w_v$ and Π_0^{1L} is increasing in w over $[w_{min}, w_v]$.

Proof of Proposition 4

The regulator prefers public leadership over simultaneous moves: $CS^{0L}(\gamma, w_{OLC}) - CS^{S}(\gamma, w_r) > 0$.

The regulator also prefers public leadership over private leadership when $\gamma < 0.4142$ because $CS^{0L}(\gamma, w_{OLC}) - CS^{1L}(\gamma, w_v) > 0$ and $CS^{0L}(\gamma, w_{OLC}) - CS^{1L}(\gamma, w_{max}) > 0$. When $\gamma > 0.4142$, it discards inducing private leadership with $w = w_{min}$ because $\Pi_0^{1L} < (>) 0$ when $w = w_{min}$ if $\gamma < (>) 0.8584$ and $CS^{0L}(\gamma, w_{OLC}) - CS^{1L}(\gamma, w_{min}) > (<) 0$ if $\gamma > (<) 0.8297$, but it may choose to induce private leadership with $w = w_{1LC}$ because $CS^{0L}(\gamma, w_{0LC}) - CS^{1L}(\gamma, w_{1LC}) > (<) 0$ if $\gamma > (<) 0.6586$.

Regulator with Social Welfare as Objective Function

To analyze the case when the regulator maximizes social welfare instead of consumer surplus, let us replace CS by SW in (21-23). Notice that SW^{0L}, SW^{1L} and SW^{S} are all concave in w. We obtain $w_{OL} = w_S = w_r$ because the first order condition for maximization of both SW^{0L} and SW^{S} holds at $w = w_r$. Also, $w_{1L} = w_{1Lf} = \frac{2a\gamma + 2a}{\gamma^2 + 5\gamma + 6}$ if $\gamma < 0.9558$ and $w_{1L} = w_{min}$ if $\gamma > 0.9558$. This is so because i) when $w = w_{1Lf}$, $\Pi_0^{1L} > 0$ and the first order condition for maximization of SW^{1L} holds, ii) $w_v < (>)\, w_{1Lf}$ if $\gamma < (>)\, 0.4142$, iii) $w_{1Lf} < (>)\, w_{min}$ if $\gamma > (<)\, 0.9558$, and iv) $w_{1Lf} < w_{max}$. Since $SW^{0L}(w_r) = SW^{S}(w_r) = SW^{1L}(w_{1Lf}) = \frac{a^2}{\gamma + 3}$, it follows that the regulator is indifferent between inducing any of the three timings if $\gamma < 0.9558$ while it is indifferent between inducing simultaneous price setting or public leadership, but prefers these options to private leadership if $\gamma > 0.9558$.

REFERENCES

Arya, A., & Mittendorf, B. (2018). Endogenous timing when a vertically integrated producer supplies a rival. *Journal of Regulatory Economics*, *54*(2), 105-123. https://doi.org/10.1007/s11149-018-9368-8

Bárcena-Ruiz, J. C. (2007). Endogenous timing in a mixed duopoly: price competition. *Journal of Economics 91*, 263-272. https://doi.org/10.1007/s00712-007-0255-5

Bárcena-Ruiz, J. C., and Sedano, M. (2011). Endogenous timing in a mixed duopoly: weighted welfare and price competition. *The Japanese Economic Review*, *62*(4), 485-503. https://doi.org/10.1111/j.1468-5876.2010.00531.x

Chen, Y., & Sappington, D. E. (2009). Designing input prices to motivate process innovation. *International Journal of Industrial Organization*, 27(3), 390-402. https://doi.org/10.1016/j.ijindorg.2008.10.008

Din, H. R., & Sun, C. H. (2016). Combining the endogenous choice of timing and competition version in a mixed duopoly. *Journal of Economics*, *118*(2), 141-166. https://doi.org/10.1007/s00712-015-0470-4

Fernández-Ruiz, J. (2024). Endogenous Timing in a Mixed Duopoly when a Public Firm Supplies its Private Competitor. *Applied Economics Letters*, 1-7. https://doi.org/10.1080/13504851.2024.2306174

Hamilton, J. H., & Slutsky, S. M. (1990). Endogenous timing in duopoly games: Stackelberg or Cournot equilibria. *Games and Economic Behavior*, *2*(1), 29-46. https://doi.org/10.1016/0899-8256(90)90012-J

Heywood, J. S., & Ye, G. (2009). Privatisation and timing in a mixed oligopoly with both foreign and domestic firms. *Australian Economic Papers*, *48*(4), 320-332. https://doi.org/10.1111/j.1467-8454.2009.00379.x

Kawasaki, A., Ohkawa, T., & Okamura, M. (2020). Endogenous timing game in a mixed duopoly with partial foreign ownership and asymmetric increasing marginal costs. *Australian Economic Papers*, *59*(2), 71-87. https://doi.org/10.1111/1467-8454.12170

King, S. P. (2013). Countervailing power and input pricing: When is a waterbed effect likely? I*nternational Journal of the Economics of Business*, *20*(3), 325-340. https://doi.org/10.1080/13571516.2013.835982

Lee, S. H., & Xu, L. (2018). Endogenous timing in private and mixed duopolies with emission taxes. *Journal of Economics*, *124*(2), 175-201. https://doi.org/10.1007/s00712-017-0565-1

Lu, Y. (2006). Endogenous timing in a mixed oligopoly with foreign competitors: the linear demand case. *Journal of Economics*, *88*(1), 49-68. https://doi.org/10.1007/s00712-006-0186-6

Matsumura, T. (2003). Stackelberg mixed duopoly with a foreign competitor. *Bulletin of Economic Research*, *55*(3), 275-287. https://doi.org/10.1111/1467-8586.00175

Matsushima, N. (2017). Expanding distribution channels. *Asia-Pacific Journal of Accounting & Economics*, *24*(3-4), 464-484. https://doi.org/10.1080/16081625.2016.1188453

Naya, J. M. (2015). Endogenous timing in a mixed duopoly model. *Journal of Economics*, *116*(2), 165-174. https://doi.org/10.1007/s00712-014-0416-2

Pal, D. (1998). Endogenous timing in a mixed oligopoly. *Economics Letters*, *61*(2), 181-185. https://doi.org/10.1016/S0165-1765(98)00115-3

Singh, N., & Vives, X. (1984). Price and quantity competition in a differentiated duopoly. *The Rand journal of economics*, *15*(4), 546-554. https://doi.org/10.2307/2555525

Wolak, F. A. (1994). An econometric analysis of the asymmetric information, regulator-utility interaction. *Annales d'Economie et de Statistique*, (34), 13-69. https://doi.org/10.2307/20075947

Zhang, Y. & Zhong, W (2015). Are public firms always less innovative than private firms? *Japanese Economic Review*, *66*(3), 393-407. https://doi.org/10.1111/jere.12047