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Debt and Endogenous Retention Rate in a Kaleckian Model of Accumulation with Productive Capacity Utilization

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*Sébastien Charles**

Abstract

The Post-Keynesian theory of debt is examined from a different perspective. First, a standard macromodel of capacity utilization and growth is presented. Then, the long-run behaviour of this model by endogenizing the stock of debt and the corporate retention rate is taken into consideration. Subsequently, it is shown that a wide range of dynamical behaviours is plausible as also the fundamental role of firms' adjustment speed to disequilibria.

Keywords: Retention Rate, Debt, Accumulation, Kaleckian Model.
Clasificación JEL: B59, E12, E22, E44.

* University of Paris 8-Saint-Denis, Department of Economics (LED-EPEH) (sebcharles92@yahoo.com).

Introducción

About twenty years ago a prolific field of research appeared in economic literature. It studies the relations between financial factors (debt ratio, interest rates and financial structure) and the stability of capitalist economies in the spirit initiated by Minsky's (1975, 1982) frameworks. Whereas Minsky did not completely develop his financial instability theory from the point of view of its formalization, some authors did not hesitate to overcome this apparent limitation. Thus, in the middle of the 1980´s eighties a new paradigm has emerged, known as Minskyan financial macroeconomics.

Taylor and O'Connell's (1985) article is, without doubt, a basic reference in this literature since it opened the way to numerous ulterior developments. These link expected profits to the rate of interest and show that, under some conditions, firms' expectations may generate instability because they increase the effects of recessions. Delli Gatti and Gallegati (1990) assess the appearance of financial fragility on the basis of a standard Minskyan diagram and the existence of lender's and borrower's risks. Recently, Nasica and Raybaut (2005) followed this approach by incorporating a government stabilization policy with a target value for investment demand. Palley (1996) focuses on the contradictory effects of debt when there exist two categories of households –borrowers and lenders– with different propensities to spend. Indeed, borrowers use obtained funds in order to increase their consumption, which has a positive impact on the level of global demand. Nevertheless, this mechanism ceases as soon as borrowers begin to pay back lenders with a low propensity to spend. Global demand declines, since those who have a higher propensity to spend now have a smaller income –they have to meet their financial commitments– whereas those who have a lower propensity to spend –lenders– take advantage of a transfer of income. In Charles (2006), the appearance of endogenous business cycles is caused by the contradictory character of debt on firms in a macroeconomic scope. Fazzari, Hubbard and Petersen (1988) have shown the essential role of cash flows for investment decisions. We know that a rise in debt produces negative effects on the level of internal resources. However, at the same time, external financing permits an increase in amount of investment generating itself new cash flows for the future. Finally, introducing this ambivalence about debt in an investment function is the cause of economic fluctuations. In the same logic, the contribution of Hubbard (1998) shows that the rate of interest has considerable consequences on accumulation through its effects on internal funds and on the access to external borrowing. The disturbing effect of interest rates is developed by Jarsulic (1990) and Charles (2005) who put forward the following idea: the capacity to absorb shocks is reduced as interest rates increase and that financial fragility becomes a reality.

In the present article, we explicitly follow the previous developments by using a neo-Kaleckian model and improving some key assumptions with regard to firms' behaviours.[1] Then, in the next section we present the main assumptions and the structure of a standard Kaleckian model of growth. Section three then obtains the short-run equilibrium values of capacity utilization, profit rate and growth rate of accumulation as well as a series of results relative to comparative static. Section four studies the equilibrium in the long run postulating an endogenous determination of debt and retention rate and, thus, analyzing financial dynamics. This permits us to show various trajectories for the economy as well as, for some conditions; the existence of a "corridor stability". Finally, in section five we draw some conclusions.

1. Structure and hypotheses

We consider a closed economy without government economic activity. The level of global demand is not enough to ensure full capacity utilization. Inflation appears when full capacity is reached. Moreover, there exists a single good that can be used for consumption and investment. This good is produced with two factors of production –capital and labour– using a fixed-coefficient technology. Technical change is not explicitly considered and, consequently, coefficients are not assumed to change.[2] Firms, producing in an imperfect goods market, set price p according to a standard mark-up on exogenous unit labour costs. Following Kalecki (1954), the mark-up is determined by dominant firms on the goods market. Price equation is written as:

$$p = (1 + m)\, wl \tag{1}$$

with w the nominal wage rate, $l = L/Y$ the labour-output ratio and $m > 0$ the mark-up.

Some simple manipulations of equation (1) allow us to determine the profit share π in nominal income pY:

$$\pi = \Pi / pY = 1 - (w / p)l = m/(1 + m)$$

[1] For a survey on Kaleckian models of growth cf. Lavoie (1992, 1995).

[2] *Cf.* Blecker (1999) for a Kaleckian model in an open economy. The reader will find in Keen (1995) a formalization of government fiscal activities in the scope of a Minsky-Goodwin model and in Lima (2000) the introduction of endogenous technical change. Downe (1987) integrates inflation to the Kalecki-Minsky model of Taylor and O'Connell (1985).

We also define the macroeconomic profit rate r as being the proportion of profits Π in nominal capital stock pK. Taking into account the standard decomposition of the profit rate contained in Kaleckian models of growth and distribution [cf. Lavoie (1992) and (1995)], r may be rewritten as described below:

$$r = \frac{\Pi}{pK} = \frac{\Pi}{pY}\frac{Y}{Y^*}\frac{Y^*}{K} = \frac{\pi u}{v} \tag{2}$$

where Y^* is potential real output corresponding to full employment, $v = K/Y^*$ is the exogenous capital-potential output ratio, and $u = Y/Y^*$ the rate of capacity utilization (when $u = 1$ economy is at full capacity).

As for the investment function, we adopt a simplified version of Lima and Meirelles (2003). Firms are not at full capacity and make investment plans according to the following desired function:

$$g^d = I/K = \alpha + \beta r - \sigma i \tag{3}$$

with g^d the desired rate of accumulation, $I = dK/dt$ investment, r the macroeconomic rate of profit, i the rate of interest, α, β and σ positive parameters.

We know that in Kalecki (1971) the desired rate of accumulation depends positively on the rate of profit. Rowthorn (1982), Taylor (1983) and Dutt (1984) keep intact this positive relationship in standard neo-Kaleckian models of growth. The negative impact of interest rates on g^d follows Dutt (1994) and finds empirical evidences in Hubbard (1998). First, an increase in i, by rising the cost of credit, deteriorates the profitability of numerous investment projects which must be abandoned. Then, it reduces the level of internal resources for future investment expenditures and it becomes harder to meet financial commitments for borrowers. Finally, it can be seen that it reduces the lenders' willingness to finance productive units with a deteriorated financial structure. The desired rate of growth also depends on a coefficient a which is an indicator of the state of confidence, independently of anything else (financial conditions, profit rate…). It is often assimilated to Keynes' (1936) animal spirits or, according to Setterfield (2003), to the state of long-run expectations.

Turning our attention to saving behaviours, we start by assuming the existence of three agents: firms, capitalists (or rentiers) and workers. Firms retain a portion of their net profits for saving purposes and distribute the rest to rentiers. We assume that capitalists' households save a constant fraction of interest receipts and spend the whole amount of distributed dividends. Moreover, in order to keep the argument simple we will assume that workers as a class do not save. Nevertheless, this does not mean that individual workers do not save. An explanation of this apparent contradiction is to be found in Foley and Michl (1999): the savings of some of them is exactly matched by that which is not saved by others. Consequently, the aggregate saving function is written in the following way:

$$g^s = S/pK = s_f\,(\mathrm{r} - \mathrm{id}) + s_i id \tag{4}$$

where S is total saving, g^s total saving as a proportion of the nominal capital stock, $0 < s_f < 1$ the retention rate or firms' propensity to save out of net profits, D the stock of debt, $d = D/pK$ the debt to capital ratio, and $0 < s_i < 1$, capitalists' propensity to save out of interest receipts.

Here, distributed dividends $(1 - s_f)(r - id)$ do not appear since we assumed that they are spent in their totality, so $s_c = 0 \Leftrightarrow c_c = 1$. Following the statistics provided by Jarsulic (1996) for the United States, the retention rate is relatively high in the long-run. However, we may consider that pure capitalists, according to their special status, have a higher propensity to save, so: $s_i > s_f$. Two arguments may be put forward to explain this value. First, rentiers would be obliged to save a high fraction of their income to avoid the vanishing of their wealth, that they could not reconstitute by working. An alternative explanation is the following: since capitalists obtain an important level of wealth they are able to spend only a small portion of their income.

2. The neo-Kaleckian model in the short run

In the short-run, the retention rate and the debt to capital ratio are given. Adjustments leading to equilibrium on the goods market ($g^d = g^s$) are obtained through changes in quantities. Thus, firms change the rate of capacity utilization u. The rule of dynamic adjustment is the following:

$$du\,/\,dt = \theta\,(g^d - g^s) \tag{5}$$

where θ is the speed of adjustment greater than zero, if $\theta = 1$ the adjustment is instantaneous.

Equation (5) simply states that production varies according to excess demand on the goods market. Some simple computations indicate that the stability condition is: $s_f - \beta > 0$. Replacing g^d and g^s by their values from (3) and (4) and the resulting expression by (1) and (2), we find the following equilibrium value for the rate of capacity utilization:

$$u = \frac{\alpha - i\left[\sigma + (s_i - s_f)d\right]}{(s_f - \beta)\pi} v \tag{6}$$

Introducing equation (6) into (2), and the resulting expression in (3) we have:

$$r = \frac{\alpha - i\left[\sigma + (s_i - s_f)d\right]}{(s_f - \beta)} \tag{7}$$

$$g = \frac{s_f(\alpha - \sigma i) + \beta(s_f - s_i)d}{(s_f - \beta)} \tag{8}$$

By computing the partial derivatives of u, r and g with respect to i and d, we show the impact of financial conditions on real economic activity:

$$\partial u / \partial i < 0 \qquad \partial r / \partial i < 0 \qquad \partial g / \partial i < 0 \tag{9}$$

$$\partial u / \partial d < 0 \qquad \partial r / \partial d < 0 \qquad \partial g / \partial d < 0 \tag{10}$$

We immediately see that the rate of interest and the debt ratio have a negative effect on the rate of capacity utilization, the global rate of profit and the growth rate of capital. Two other fundamental results of standard neo-Kaleckian models concern the consequences of variables regarding the distribution of income on u and g:

$$\partial u / \partial \pi = -\frac{\alpha - i[\sigma + (s_i - s_f)d]}{(s_f - \beta)\pi^2} v < 0 \tag{11}$$

$$\partial g / \partial \pi < 0 \tag{12}$$

Equation (11) indicates that the model is stagnationist in essence. This means an increase in the real wage, i.e. a fall in the share of profit, with $\pi = 1 - (w/p)l$, causes a rise in the rate of capacity utilization. This apparent paradox is explained by the fact that a rise in the real wage tends to redistribute income from rentiers who have a small propensity to spend to workers with a propensity to spend equals to unity. This has a positive impact, increasing consumption and global demand and, consequently, the rate of capacity utilization. With regard to the effect of w/p on the growth rate of capital, three cases may exist. If an increase in the real wage has a positive impact on g we are in the presence of a wage-led economy. Inversely, Bhaduri and Marglin (1990) show that a rise in the real wage may cause a fall in the rate of accumulation; the economy is said to be profit-led. The last possibility is to say that a variation in the level of real wage has no impact on investment since it creates contradictory forces which counterbalance each other. This is the result obtained through expression (12). A quick inspection of (2) shows that a rise in the real wage causes an increase at the rate of capacity utilization and a fall in the profit rate in the same time. Both effects strictly counterbalance each other which leaves the value of the rate of accumulation unchanged. As shown by equation (3), if r does not change this involves the stability of the rate of accumulation and: $\partial g/\partial\pi = 0$.

3. Long run dynamics

In the long-run analysis the debt to capital ratio and firms' retention rate are endogenous variables. Taking the derivatives of the debt to capital ratio with respect to time and assuming away inflation, we find:

$$\dot{d} = \dot{D}/K - dg \tag{13}$$

We also assume, according to Asada (2001) and Charles (2005), that if investment I exceeds the level of retained earnings $s_f(\Pi - iD)$ firms get into debt, so: $\dot{D} = I - s_f(\Pi - iD)$. Consequently, the dynamics of debt is written as:

$$\dot{d} = (1-d)g - s_f(r - id) \tag{14}$$

The variation of the retention rate is the following:

$$\dot{s}_f = \eta(d - \bar{d}) \tag{15}$$

with $\bar{d}$ the level of the debt to capital ratio that firms consider prudent and $\eta > 0$ a coefficient accounting for the speed of adjustment of firms to lags between d and its target value.

If $\bar{d} < d$ firms' managers think that they are in a rather risky situation since the level of retained earnings tends to decline. They react by increasing the retention rate in order to counterbalance the fall in net profits. Here we introduce a mechanism, emanating from managerial decisions, which tends to preserve firms' financial autonomy.[3] Moreover, the rise in retention rate mechanically increases non distributed profits and the capacity to meet financial commitments with respect to debtors. This rise also improves the financial structure and diminishes the risk of speculative and Ponzi financing and the probability of bankruptcy. Consequently, these elements clearly show another point: firms accept to increase the target debt ratio to the extent that the retention rate also increases. Assuming a linear relation for the sake of simplicity we have:

$$\bar{d} = \phi_o + \phi_1 s_j \qquad \text{with } \phi_i > 0 \tag{16}$$

Replacing the target debt to capital ratio (16) into equation (15) and the equilibrium values of r and g from expressions (7) and (8) into (14), we obtain the following non-linear dynamic system in continuous time:

$$= \eta(d - \phi_0 - \phi_t s_f) \tag{17}$$

$$\dot{d} = (1-d)\left[\frac{s_f(\alpha - \sigma i) + \beta(s_f - s_i)d}{s_f - \beta}\right] - s_f\left[\frac{\alpha - i[\sigma + (s_i - s_f)d]}{s_f - \beta} - id\right] \tag{18}$$

[3] Note that there is no reaction from shareholders. For instance, we could also introduce a conflict mechanism between managers and shareholders. Here, we focus on firms' reaction and leave this kind of formalization to further developments.

The behaviour of the system of differential equations (17) and (18), despite their non linear form, may be found by the construction of a standard phase diagram. In the long run equilibrium, the endogenously determined retention rate and debt-capital ratio have to be constant, so we have the following equality: $\dot{s}_f = \dot{d} = 0$. Isoclines are equal to:

$$d \Big|_{\dot{s}_f=0} = \phi_0 + \phi_1 s_f \tag{19}$$

and:

$$d \Big|_{\dot{d}=0} = \frac{s_f\left[\alpha - i(\sigma + s_i - \beta)\right] + \beta s_i}{(s_i - s_f)\beta} \tag{20}$$

Computing the derivatives of the debt-capital ratio with respect to firms' propensity to save we find the following equations:

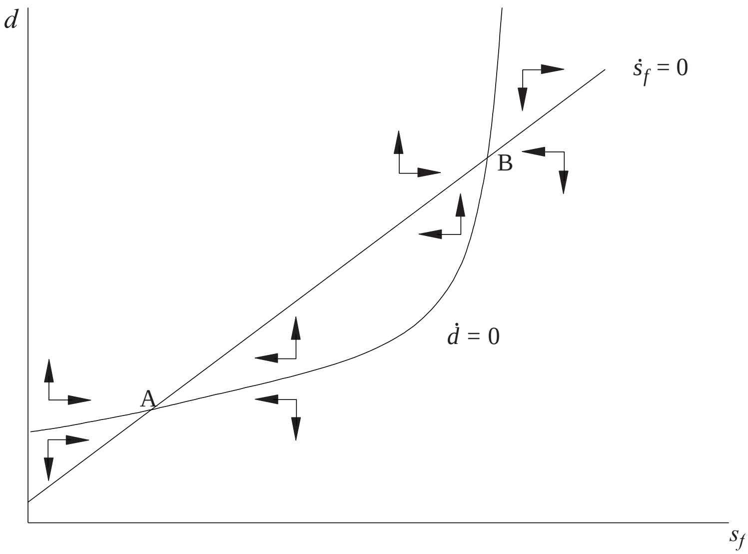
$$\left.\frac{\partial d}{\partial s_f}\right|_{\dot{s}_f=0} = \phi_1 > 0 \tag{21}$$

$$\left.\frac{\partial d}{\partial s_f}\right|_{\dot{d}=0} = \frac{s_i\beta\left[\alpha - i(\sigma + s_i - \beta)\right]}{\left[(s_i - s_f)\beta\right]^2} > 0 \tag{22}$$

$$\left.\frac{\partial^2 d}{\partial s_f^{\,2}}\right|_{\dot{d}=0} = \frac{2s_i\beta^2\left[\alpha - i(\sigma + s_i - \beta)\right]}{\left[(s_i - s_f)\beta\right]^3} > 0 \tag{23}$$

We immediately see that the shape of $\dot{s}_f = 0$ is always positive. Some difficulties appear when dealing with $\dot{d} = 0$ Recalling the logical constraint on saving behaviours $s_i - s_f > 0$, we see that the denominator in expressions (22) and (23) is always positive. As for the numerator, it remains positive as long as animal spirits, or the state of confidence, is high enough. Moreover, the relatively low value of the interest rate should also permit the possibility of respecting the positivity of numerator. The phase diagram drawn in Figure 1 shows, due to the presence of non-linearities, the existence of multiple equilibria.

Figure 1
Phase diagram for debt and retention rate



These results may also be found algebraically by studying the Jacobian matrix associated with equations (17) and (18):

$$J_{11} = -\ \eta\phi_1 < 0$$

$$J_{12} = \eta > 0$$

$$J_{21} = \frac{\alpha - i(\sigma + s_i - \beta) + (s_i - \beta)(1-d)}{(s_f - \beta)^2}$$

$$J_{22} = \frac{\beta(s_i - s_f)(2d-1) - s_f\left[\alpha - i(\sigma + s_i - \beta)\right]}{(s_f - \beta)}$$

As shown in Appendix 1, there exist various possibilities leading to three kinds of equilibrium: saddle point equilibrium, stable and unstable spiral points. Then, it is clear that equilibrium A is a saddle point and B is a spiral point. The stability of B depends on the sign of $Tr\,(J) = J_{11} + J_{22}$. We are now able to study the impact of financial variables on the trace of J. We have:

$$\frac{\partial Tr(J)}{\partial i} = \frac{s_f(\sigma + s_i - \beta)}{(s_f - \beta)} > 0$$

An increase in the rate of interest generates instability since it tends to increase J_{22} and to make positive the trace of J. Thus, it involves the appearance of case 1 (cf. Appendix 1). The passage from A to B may be explained in the following way: if the debt-capital ratio increases firms have no other choice than to raise the retention rate, i.e. to distribute a smaller amount of dividends to shareholders. This reaction permits maintaining retained earnings to some level and to preserve firms' capacity to meet financial commitments.

Now an essential point has to be explained. Equilibrium B is precarious because it heavily depends on the behaviour of firms through the parameter η. We already know that this latter stands for the speed of adjustment to which firms react when there is a lag between the effective level of d and the target debt-capital ratio considered as being secure. If firms take more time to react, the parameter η is low and we have $Tr\,(J) > 0$. At some parameter value $\eta_0 = J_{22}/\phi_1$ –involving $Tr\,(J) = 0$– a Hopf bifurcation occurs and it appears as a limit cycle surrounding the equilibrium (s_f*, d*) (cf. Appendix 2 for a proof). Numerical simulations in figures 2 and 3 clearly show the appearance of an unstable limit cycle. We set the following initial conditions to: $s_f(0) = 0.7$ and $d\,(0) = 1.4$, with: $\phi_0 = 0.5$, $\phi_1 = 1.2$, $\sigma = 0.3$, $\alpha = 0.03$, $\beta = 0.3$, $i = 0.02$ and $s_i = 0.8$.

The more firms take time to react to changes in retained earnings –i.e. the smaller η is– the more risky the situation. It is now obvious that when η is low the probability to face instability becomes higher.

Figure 2
Corridor stability for η = 0.2

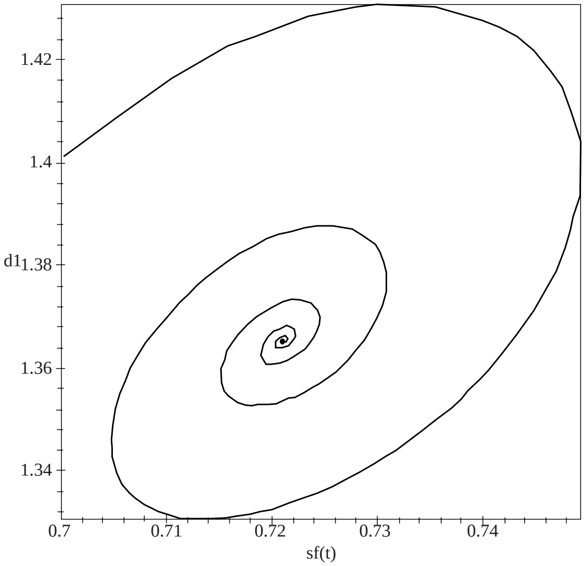
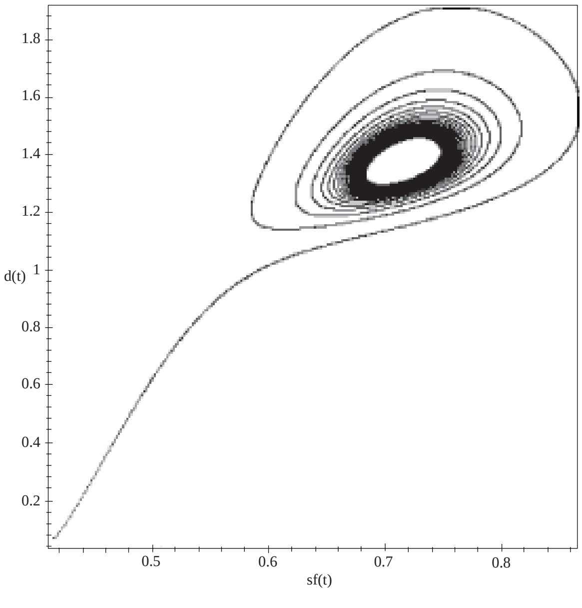


Figure 3
Unstable Cycles for η = 0.113



Moreover, the dynamics of this unstable limit cycle show that the economic system is only capable of absorbing "small" shocks and that it is vulnerable to larger perturbations. This case corresponds to the "corridor stability" analysed by Leijonhuvfud (1973) and formalized, among others, in Semmler and Sieveking (1993). This leads to the conclusion, not very optimistic, according to which external financing of investment and its impact on the behaviour of firms with regard to the retention rate plays a very destabilizing role.

Conclusion

In this article we developed a neo-Kaleckian model of capacity utilization and growth in which the level of debt-capital ratio and the retention rate are endogenous in the long run. We reached several conclusions concerning the impact of financial conditions and the real wage on the short run equilibrium values of the rate of capacity utilization, the global profit rate and the growth rate of capital. It appears that the economy is demand-led or stagnationist since an increase in the real wage leads to a rise in the rate of firms' productive capacity utilization. Moreover, this result is standard in the scope of Kaleckian models of growth and distribution as shown by the contributions of Rowthorn (1982), Dutt (1984), Taylor (1983, 1991) and Lavoie (1992, 1995).

As for the dynamics in the long run, we basically showed that the conditions for stability are rather difficult to maintain and that firms have a very important role to play in the existence of unstable cycles. More precisely, a "disguised conflict" between shareholders and managers would be at the origin of economic and financial instability; the former putting pressure on the latter to received a larger amount of dividends. Besides, when the debt-capital ratio is high stability may be maintained only by assuming low values for the rate of interest. This condition is very precarious since, according to Minsky (1986), the deterioration of firms' financial structures often involves an increase in interest rates. Consequently, this model develops the conclusions of the financial macroeconomics *à la* Minsky by endogenizing the saving behaviour of firms in a Kaleckian scope.

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Appendix 1

Stability analysis

Studying the Jacobian matrix of (17) and (18) leads to four possibilities:

$$(1)\begin{bmatrix} - & + \\ - & + \end{bmatrix} \qquad (2)\begin{bmatrix} - & + \\ - & - \end{bmatrix} \qquad (3)\begin{bmatrix} - & + \\ + & - \end{bmatrix} \qquad (4)\begin{bmatrix} - & + \\ + & + \end{bmatrix}$$

Case 1: If *Det* (J) > 0, the stability depends on the sign of *Tr* (J) which is ambiguous. If *Det* (J) < 0, the equilibrium is a saddle point.
Case 2: We have *Det* (J) > 0 and *Tr* (J) < 0, the equilibrium is stable.
Case 3: If *Det* (J) > 0, the equilibrium is stable since *Tr* (J) < 0. Otherwise, we have a saddle point equilibrium.
Case 4: The equilibrium is a saddle point since *Det* (J) < 0.

Appendix 2

Hopf bifurcation theorem, *Wiggins (1990)*

Let us assume the following differential system: $dx/dt = f(x,\eta)$ where $x \in \mathrm{R}^n$ with $n \geq 2$ and η a parameter. This system has a solution written: $x^*(\eta)$. Let J the Jacobian matrix of the system evaluated at point $x^*(\eta)$. If J has a pair of complex conjugate roots $\lambda(\eta), \bar{\lambda}(\eta)$ and that the following conditions are satisfied for a critical value of η_0:

1) Re $\lambda(\eta_0) = 0$, the real part of the characteristic roots at $\eta = \eta_0$.
2) Im $\lambda(\eta_0) \neq 0$, the imaginary part at $\eta = \eta_0$.
3) $\left.\dfrac{d\,Re\,\lambda(\eta)}{d\eta}\right|_{\eta=\eta_0} = v \neq 0.$
4) J has no other roots with zero real parts.

In this case, there exists a limit cycle for each value of η near h_0. If $v > 0$ and $\eta > h_0$ or $v < 0$ and $\eta < h_0$ we have a stable limit cycle (the Hopf bifurcation is supercritical). On the other hand, if $v < 0$ and $\eta > h_0$ or $v > 0$ and $\eta < h_0$ we have an unstable limit cycle (the Hopf bifurcation is subcritical).
The characteristic roots of (17) and (18) are given by:

$$\lambda_{1,2} = \frac{Tr(J) \pm \sqrt{[Tr(J)]^2 - 4Det(J)}}{2}$$

If $\eta = \eta_0$, real and imaginary parts are written, respectively, $\lambda_{1,2} = Tr(J) / 2 = 0$ and $\lambda_{1,2} = \pm i\sqrt{Det(J)} \neq 0$ with $i = \sqrt{-1}$. Since: $d\mathrm{Re}\lambda(\eta) / d\eta = v = -\phi_1 / 2 < 0$ with $\eta > \eta_0$. Thus, there exists an unstable limit cycle around the equilibrium values (s_f*, d*).