

## Analysis of the Mexican Peso-US Dollar exchange rate volatility through stochastic modeling

## Análisis de la volatilidad del tipo de cambio peso-dólar a través de modelos estocásticos

Francisco López-Herrera\*

Profesor Titular C, T.C., División de Investigación, Facultad de Contaduría y Administración, Universidad Nacional Autónoma de México.  
Correo electrónico: francisco\_lopez\_herrera@yahoo.com.mx. ORCID <https://orcid.org/0000-0003-2626-9246>

### ABSTRACT

We analyze the Mexican peso-US dollar exchange parity the volatility using three models in order to evaluate the exchange rate risk performance and the these models capability of modeling such risk. The period June 21<sup>st</sup>, 2017 to June 21<sup>st</sup>, 2022 is marked by singular events that pressed this exchange rate parity, making it of great interest to study the tools that can be used to measure this uncertainty besides the study of the exchange rate uncertainty itself. The arrival of a left-wing politician to the presidency was an unprecedented event. At the beginning of 2020 the price war in the oil market and the emergence of the COVID-19 pandemic had important effects in the world economy. Finally, the war in Ukraine added pressures to the international trade of raw materials, inducing an inflationary episode with bad expectations about the economic performance around the world. The exchange rate scenarios posed by the aforesaid events provide us with a natural context to test which of the posited volatility models offers a better fit. Unanimously, the results under the three models suggest that during the first year of the President López Obrador period the exchange rate uncertainty was reduced compared to previous levels, returning again to those minor levels after the forced closure of economic activities due to the emergence of COVID-19, even though the exchange fluctuation band has been slightly higher than before the pandemic and the war on East Europe.

### RESUMEN

Analizamos la volatilidad de la paridad cambiaria peso-dólar mexicano utilizando tres modelos con el fin de evaluar el desempeño del riesgo cambiario y la capacidad de estos modelos para modelar dicho riesgo. El periodo comprendido entre el 21 de junio de 2017 y el 21 de junio de 2022 está marcado por eventos singulares que presionaron esta paridad cambiaria, por lo que es de gran interés estudiar las herramientas que se pueden utilizar para medir esta incertidumbre además de estudiar la propia incertidumbre cambiaria. La llegada de un político de izquierda a la presidencia fue un hecho sin precedentes. A principios de 2020, la guerra de precios en el mercado petrolero y la aparición de la pandemia de COVID-19 tuvieron efectos importantes en la economía mundial. Finalmente, la guerra en Ucrania sumó presiones al comercio internacional de materias primas, induciendo un episodio inflacionario con malas expectativas sobre el desempeño económico en todo el mundo. Los escenarios cambiarios planteados por los eventos mencionados nos proporcionan un contexto natural para probar cuál de los modelos de volatilidad postulados ofrece un mejor ajuste. De manera unánime, los resultados bajo los tres modelos sugieren que durante el primer año del periodo del presidente López Obrador la incertidumbre cambiaria se redujo en comparación con niveles anteriores, regresando nuevamente a esos niveles menores tras el cierre forzoso de actividades económicas por la aparición del COVID-19, a pesar de que la banda de fluctuación cambiaria ha sido ligeramente superior a la anterior a la pandemia y la guerra en Europa del Este.

*Received: September/19/2023*

*Accepted: November/03/2023*

*Posted: January/31/2024*

#### Keywords:

| Foreign exchange |  
| Mexican peso | Exchange  
rate | Volatility models |  
| COVID-19 |

#### Palabras clave:

| Divisas | Peso mexicano |  
| Modelos de volatilidad |  
| Tipo de cambio |  
| COVID-19 |

#### JEL Classification |

Clasificación JEL |

C52, C58, F31



Esta obra está protegida  
bajo una Licencia  
Creative Commons  
Reconocimiento-  
NoComercial-  
SinObraDerivada 4.0  
Internacional

## INTRODUCCIÓN

In general, as view by the people of a country, exchange rates are the prices of foreign currencies. The demand for foreign currencies obeys to several causes, the obvious are the needs that arise out the international trading relationships, so as that produced by cross-border financial transactions and, maybe in a minor extension for some countries, the divisas required by tourism activities. So, a foreign exchange market has a very important function, that is the transference of purchasing and financing power among individuals, enterprises and organizations of different nations.

In a more formal vein, there are different theories that relates the exchange rates to the performance of economic and financial variables, the main considered variables are the level prices, interest rates and current account balances. In turn, these variables are also related to other economic forces, both local and foreign, in such a way that the exchange rate not only influences other economic and financial variables, but is also influenced by them, that is, there are feedback effects between the dynamics of the exchange rate and the general behavior of the economy. On the other hand, the uncertainty about the performance of the domestic currency regards to the foreign currencies can have negative effects into the expectations and behavior of economic agents. So, exchange rates and their volatilities have become one of the main indicators of both current and expected economic performance, thus explaining the great interest that can be observed in the media so as in the academic world.

In the presidential elections of July 2018, the one that has become the main party triumphs, accessing the presidency of the Mexican Republic for the first time, after several attempts, a leftist party, headed by Andrés Manuel López Obrador, widely known as AMLO, who took office on December 1st, 2018. Noticeably different from what was normally the case with previous rulers, sometimes as a part of the attacks led by political opponents of his government, newspaper columns have made an extra-effort to prioritize the broadcasting of pessimistic perspectives regarding the country's economic performance. It is also worth to note that stopping the construction of the new airport in the Mexican capital in the location chosen by the previous administration unleashed a wave of negative opinions that could have had effects on the mood of economic agents.

As has been widely documented, the complicated world environment caused first by the oil price war in the early 2020's months, and almost immediately by the effects of the COVID-19 pandemic, induced expectations of a poor performance in the economies of the whole world, expectations that worsened with the surge of the war in Europe. Such context provides a stressed environment suited to appraise the capability of econometric models to capture the uncertainty around the exchange rate, so the new "superpeso" period is not taken into account. In particular, it is relevant to know the available tools to measure the Mexican peso/US dollar exchange risk, because it is the main foreign currency for the Mexican economy, because the analysis is intended to take advantage of the stressed period.

At least, since the Markowitz (1952) framework for portfolio analysis, the standard deviation of the returns has been regarded as a conventional measure able to take in account the volatility/risk produced by the change on the prices of the financial assets. Besides portfolio decisions, there are another related uses of that risk measure. Along the time, several volatility features have been widely recognized. Its exhibited time-varying behavior has been summarized as a result of several outstanding stylized facts that include, mainly, volatility clustering, heavy tails and time serial dependence.

Into the context of financial econometrics time series analysis two approaches have evolved as the main devices to deal with these features of the volatility. The first approach is the GARCH modeling with its corresponding avenue of research. Being the ARCH model of Engle (1982) the first seminal paper that uses

past observations to provide estimation of the changing volatility of a time series, in the original case being the UK inflation rate. The resulting growing plethora of GARCH models up to date, and their intensive use, can be regarded as evidence of the scholars and practitioners' concerns about the volatility of financial risky assets. The other relevant approach to volatility modeling is the stochastic volatility (SV) models, being regarded the Taylor's (1982) model as the pioneering work on this family of models. As an alternative to the GARCH modeling framework, stochastic volatility models follows a different route. In stochastic volatility modeling the variance of returns is specified as realizations of a latent stochastic process, i.e., the volatility is a function of a random variable not directly observed.

So, this paper contributes to the relevant literature, analyzing the volatility of the Mexican Peso/US dollar exchange rate, taking advantage of a period marked with critical events, being the main one the irruption of the COVID-19 pandemics in the midst of a peculiar internal political context, not seen before in Mexico. The triumph of Andrés Manuel López Obrador at the polls was the beginning of a new political scenario characterized for its social policies, but its performance on several economic areas has not been appraised, partially because the presidential period is not over yet and also because an extension of it has elapsed between problems, such as the aforementioned, the analysis of which has been preemptory. This paper tries to shed some light about the performance of the Mexican peso against the US dollar, as widely known, the currency of the main economy in the world and the neighbor and principal trade partner of Mexico. The currency risk involved in the Mexican peso/USD exchange rate is very important for Mexican economic agents, so, this paper contributes to enhance our understanding of the relationships between both currencies in the face of complex scenarios for both countries. By modeling the exchange rate uncertainty through three stochastic volatility models, this paper also shed light about the convenience of the alternative specifications used. That is, our work also provides evidence about which can be the best tool for modeling that uncertainty.

The remainder of the paper is organized in the following way. After this introduction, a related literature review is offered. Next, our modeling framework is depicted in detail, following by the modeling and analysis of the volatility of the exchange rate of our concerns. Finally, we offer a concluding remarks section.

## **I. LITERATURE REVIEW**

The standard deviation, commonly referred as volatility in the financial parlance, is a measure of the variation of financial returns, caused by the movement of the assets' prices, that has been in the scene of the economic and financial decisions related to portfolio allocation, assets pricing and financial risk management. Its importance has been stated and sustained in early papers as Markowitz (1952, 1957, 1959), Lintner (1965), Mossin (1966), Sharpe (1964), Treynor (1961, 1962), Black & Scholes (1973), and other influential scholars that influenced the world of the practitioners.

The Engle (1982) paper has become a milestone in the modeling of volatility, his Autoregressive Conditional Heteroscedastic (ARCH) model is a very successful attempt to capture the volatility clustering normally observed in the evolution of the volatility of the financial returns. This was one the main contribution of Engle to the econometrics of time series that lead to the Nobel Price granted to him in 2003. The stochastic volatility (SV) models have emerged going by its own way, being one of its distinctive features, as quoted by So *et al.* (2021), that it does not use the concept of conditional variance to estimate volatility based on past squared innovations and conditional variances, as GARCH framework does, rather SV models posit a latent variable following a stochastic process in such a way that allows the evolution of volatility in a stochastic path.

As Kim *et al.* (1998) point out, the stochastic volatility process in a SV model is assumed as stationary and the empirical version of the canonical SV model posits that the returns of a financial assets are explained for the log-volatility multiplied by a constant scaling factor than can be thought as the modal instantaneous factor, being the resultant product affected by an error or random shock. At the same time, log-volatility process can have a drift term determining the mean level around which the log-volatility process wanders through the time.

Lagged log-volatility term(s) are included associated to the their respective assumedly stationary coefficients, for example, if an AR(1) process is sufficient to capture the lagged log-volatility influence on the actual log-volatility, then according with the stationarity assumption, the absolute value of the corresponding coefficient must be less than one. Stationarity assumptions are stated for other versions of the time-varying volatility stochastic process. A point to note is that this kind of coefficients give account of the persistence degree of past volatility.

Into the daily foreign exchange markets operations, the estimation of daily uncertainty is an issue of major importance because it enables to relevant agents to make decisions on hedging against negative scenarios of the market quotations, and the availability of sound tools is always welcome by the market participants. Unfortunately, some difficulties in estimating SV models have made them less popular than GARCH type options. The main issue against the estimation of SV models has been for a long the troublesome computing of the Maximum Likelihood Estimator, while for GARCH models the task is relatively easy, for SV models it is quite complicated and time consuming.

Taking advantage of the fact that, although not linear in its specification, the simplest version of the SV model can be linearized allowing the use of known results from statistical theory related to stochastic processes, Kim *et al.* (1998) propose its estimation by means of Markov Chain Monte Carlo (MCMC) methods instead of the Kalman filter technique previously suggested by Harvey *et al.* (1994), according which the SV model parameters can be estimated maximizing the corresponding quasi-likelihood function. For the estimation procedure Kim *et al.* suggest two options based on the Metropolis-Hastings and the Gibbs sampling algorithms. To illustrate the first algorithm, they model the volatilities of the exchange rate of the dollar against the pound sterling, the German mark, the Japanese yen and the Swiss franc, finding slow convergence despite the large number of iterations (1 million). To overcome the convergence problem, they propose an improved version of the algorithm, achieving substantial improvements that facilitate the estimation of even more complex stochastic volatility models. Furthermore, in their study they provide for the first time a comparative analysis of the SV and GARCH models. In general, it can be said that the importance of that deep study lies in setting the research agenda on SV models.

Following Hamilton (1989) and Taylor (2005), López-Herrera *et al.* (2011) modeled the stochastic volatility of the Mexican Peso/US dollar exchange rate as a process determined by a Markov chain with a state with low volatility and other state with high volatility; the parameters of the model were estimated maximizing the likelihood function assuming a normal mixture in the probabilistic density distribution of the errors. In a more recent paper, Avilés & Flores (2021) compares the forecasting efficiency of stochastic volatility models of the Mexican peso/US dollar exchange rate following AR(1) and AR (2) log-volatility process, GARCH (1,1), GARCH (1,2) and EGARCH models; do not finding conclusive evidence favoring a model.

At the other hand, as widely known, the COVID-19 pandemic emergence had significant impacts on the stock and exchange markets of all the world's economies and several studies have focused on evaluating the consequences of the pandemics on exchange rates. Devpura (2021) detected significant influence of COVID-19 data on the Euro/US dollar exchange rate during March of 2020. Chuanjian *et al.* (2021) have studied comparatively the effects of the crisis on the relations between the Yuan and the Euro as well as those

corresponding to the latter and the US dollar. According to their findings, there were negative effects on both the short-term and long-term equilibrium relationship between the currency pairs they analyzed. Erer (2023) studies the effects of news on exchange rate volatilities of developed and emerging economies, including Mexico. His findings provide evidence that fakenews had a negative impact increasing the exchange rate volatility.

Using Bayesian econometric methods, Pasiouras & Daglis (2020) analyzed the intraday exchange rates of the dollar against the Australian dollar (AUD), the Swiss Franc (CHF), the Euro (EUR), the British pound or Sterling (GBP), and the Hong Kong dollar (HKD), finding that confirmed cases of COVID-19 pressured currency movements. Also analyzing intraday data, Narayan *et al.* (2020) found influence of COVID-19 on exchange rates. Narayan (2020) analyzes the Yen/US dollar hourly exchange rate from July 1<sup>st</sup>, 2019 to September 4<sup>th</sup>, 2020, concluding that COVID-19 change the resistance of the Yen to shocks. Beckmann & Czudaj (2022) analyzes the impact of the pandemic on exchange rates based on a set of survey forecasts for more than 50 currency pairs, concluding that foreign exchange markets take expected policy effects as an important determinant of future developments into account while expectations for minor currencies react stronger to response policies.

According to Corsetti and Marin (2020) the irruption of COVID-19 conduced to large foreign exchange movements in emerging markets, increasing the foreign exchange and the pattern of capital outflow in an extension notably larger when compared to the 2007-2008 financial crisis. Villarreal-Samaniego (2021) detects significant co-movements between the exchange rates of the Brazilian, Colombian, Mexican, Russian and South African currencies regards the US dollar and several measures of the severity of COVID-19 pandemics throughout the first quarter of 2020, concluding that those movements were consequence of the financial turmoil induced by the spread of the pandemics around the world and the price war occurred into the oil market. Agosin and Díaz (2023) have found evidence that the volatility of the exchange rate is higher in the emerging economies in comparison with the advanced ones, and that volatility is negative correlated with the foreign capital flows. Nevertheless, it is worth to note that Giofré and Sokolenko (2023) have detected a global declining effect of the exchange rate volatility on the financial markets. So we have that not everything is bad news.

## II MODELING FRAMEWORK

Following Chan & Hsiao (2014), the exchange rate observation in time  $t$ ,  $y_t$ , is given by a nonlinear state space model in which the measurement equation is nonlinear in the state as follows

$$y_t = e^{\frac{1}{2} h_t} \varepsilon_{it} \quad (1)$$

$t = 1, \dots, T$ , and  $\varepsilon_t \sim N(0,1)$ . The conditional variance of  $y_t$  is  $Var(y_t | h_t) = e^{h_t}$ , the state  $h_t$  is also known as the log-volatility that evolves according to the AR(1) stationary process

$$h_t = \mu_h + \phi_h (h_{t-1} - \mu_h) + \zeta_t \quad (2)$$

$t = 2, \dots, T$ , and  $\zeta_t \sim N(0, \sigma_h^2)$  independently of  $\varepsilon_t$  at all leads and lags. According with the stationarity assumption  $|\phi_h| < 1$ , and the states can be initialized with the stationary distribution of the process, i.e.,  $h_1 \sim N(\mu_h, \sigma_h^2 / (1 - \phi_h^2))$ . The specification of the model includes the prior distributions

$$\mu_h \sim N(\mu_{h0}, V_{h0}), \quad \phi_h \sim N(\phi_{h0}, V_{\phi_h}) I(|\phi_h| < 1), \quad \sigma_h^2 \sim IG(V_h, S_h), \quad (3)$$

$I(\cdot)$  is the indicator function and  $IG(\cdot, \cdot)$  is the inverse-gamma distribution. Independence of these prior distributions is also assumed, that is,  $p(\mu_h, \phi_h, \sigma_h^2) = p(\mu_h) p(\phi_h) p(\sigma_h^2)$ .

Chan & Hsiao (2014) point out the challenge posed by this more complex model due to the fact that the joint conditional density of the states,  $\mathbf{h} = (h_1, \dots, h_T)'$ , given the model parameters and the data, is high-dimensional and non-standard rendering very cumbersome the Bayesian estimation via Markov Chain Monte Carlo (MCMC) methods, in sharp contrast with a gaussian linear state space model. To overcome this estimation issue, the Kim *et al.* (1998) auxiliary mixture sampler can be instrumented because this sampler allows to approximate the nonlinear stochastic volatility model through a mixture of linear Gaussian models, whose estimation is standard. The procedure starts transforming equation (1) to get

$$y_t^* = h_t + \varepsilon_t^*, \quad (4)$$

$y_t^* = \log y_t^2$  and  $\varepsilon_t^* = \log \varepsilon_t^2$ . Setting  $y_t^* = \log(y_t^2 + c)$  for a small constant  $c$ , e.g.  $c = 10^{-4}$ , is a common practice in order to avoid numerical issues when  $y_t$  is close to zero.

Although (2) and (4) conforms a state space model that is linear in  $h_t$ ,  $\varepsilon_t^*$  has not a Gaussian distribution but it follows a  $\log\text{-}\chi^2_1$  distribution, so a further step of the auxiliary mixture sampling is required to approximate the density function of  $\varepsilon_t^*$  before the conventional procedure to fit linear Gaussian state space models can be applied.

Such approximation is satisfied by the Gaussian mixture

$$f(\varepsilon_t^*) \approx \sum_{i=1}^n p_i \varphi(\varepsilon_t^*; \mu_i, \sigma_i^2), \quad (5)$$

where  $\mu_i$ ,  $\sigma_i^2$  and  $p_i$  are, respectively, the mean, the variance and the probability of the  $i$  is mixture component,  $n$  is the number of components.

According to Chan and Hsiao (2014), the mixture density in (5) can equivalently written in terms of the auxiliary random variable  $s_t \in \{1, \dots, n\}$ , whose role as the mixture component indicator gives the name to the auxiliary sampler, providing us with a linear Gaussian model conditional on  $s_t$ :

$$(\varepsilon_t^* | s_t = i) \sim N(\mu_i, \sigma_i^2), \quad (6)$$

$$\mathbb{P}(s_t = i) = p_i. \quad (7)$$

Kim *et al.* (1998) propose a seven-component Gaussian mixture for approximating the  $\log\text{-}\chi^2_1$  distribution that provides fixed values of the parameters, as given in Chan & Hsiao (2014, Table 6.1, p. 159), not requiring additional computation time in the estimation. By means of this mixture approximation, the model (2) and (4) is conditionally Gaussian linear given the component indicators  $\mathbf{s} = (s_1, \dots, s_T)'$  and the MCMC simulation techniques for fitting linear Gaussian state space models become available.

Defining  $\mathbf{y}^* = (y_1^*, \dots, y_T^*)'$  so as  $\mathbf{h}$ ,  $\boldsymbol{\zeta}$  and  $\boldsymbol{\varepsilon}^*$ , and using a sample from the joint posterior distribution  $p(\mathbf{h}, \mathbf{s}, \mu_h, \phi_h, \sigma_h^2 | \mathbf{y})$ , conventional Bayesian analysis can be carried out. A Gibbs sampler can be used to obtain the posterior draws from the mixture approximation iterating through.

1.  $p(\mathbf{h} | \mathbf{y}^*, \mathbf{s}, \mu_h, \phi_h, \sigma_h^2)$ ;
2.  $p(\mathbf{s} | \mathbf{y}^*, \mathbf{h}, \mu_h, \phi_h, \sigma_h^2) = p(\mathbf{s} | \mathbf{y}^*, \mathbf{h})$ ;
3.  $p(\mu_h | \mathbf{y}, \mathbf{h}, \mathbf{s}, \phi_h, \sigma_h^2) = p(\mu_h | \mathbf{h}, \mathbf{s}, \phi_h, \sigma_h^2)$ ;
4.  $p(\phi_h | \mathbf{y}, \mathbf{h}, \mathbf{s}, \mu_h, \sigma_h^2) = p(\phi_h | \mathbf{h}, \mu_h, \sigma_h^2)$ ;
5.  $p(\sigma_h^2 | \mathbf{y}, \mathbf{h}, \mathbf{s}, \mu_h, \phi_h) = p(\sigma_h^2 | \mathbf{h}, \mu_h, \phi_h)$ .

To estimate the conditionally linear Gaussian state space model represented in (2), (4), (6) y (7), given the prior distributions in (3), Chan & Hsiao (2014) propose a precision sampler, an algorithm based in the fact that the inverse of the covariance matrix, i.e. the precision matrix, of  $p(\mathbf{h} \mid \mathbf{y}^*, \mathbf{s}, \mu_h, \phi_h, \sigma_h^2)$  is a band matrix containing only a small number of nonzero elements along a diagonal band, so the computation time required by the iterations through the Gibbs sampler can be significantly reduced in comparison with the computation involving full matrices. The algorithm includes an independence-chain Metropolis-Hastings step. To estimate the SV model with the constant conditional mean,  $\mu$ , an extra block is included to sample from the conditional distribution  $p(\mu_h \mid \mathbf{y}, \mathbf{h})$ , and the Gibbs sampler is modified using  $y_t - \mu$  instead  $y_t$ .

Chan & Hsiao (2014) consider to extend the autoregressive stochastic volatility (SV) model given by (1) and (2) in order to allow persistence in the shocks via a moving average process in the errors of the measurement equation, a device useful to test efficiency hypothesis. Under a MA(q) model specification we have:

$$y_t = \mu + u_t, \quad (8)$$

$$u_t = \varepsilon_t + \psi_1 \varepsilon_{t-1} + \dots + \psi_q \varepsilon_{t-q}, \quad (9)$$

$$h_t = \mu_h + \phi_h (h_{t-1} - \mu_h) + \zeta_t, \quad (10)$$

$\varepsilon_t \sim N(0, e^{h_t})$  and  $\zeta_t \sim N(0, \sigma_h^2)$  are independent of each other,  $\varepsilon_0 = \varepsilon_{-1} = \dots = \varepsilon_{-q+1} = 0$ . According with the invertibility conditions, the roots of the characteristic polynomial associated with the MA coefficients  $\boldsymbol{\psi} = (\psi_1, \dots, \psi_q, 0)'$  lie outside the unit circle. Also  $|\phi_h| < 1$  is assumed and the states are initialized with  $h_1 \sim N(\mu_h, \sigma_h^2 / (1 - \sigma_h^2))$ . Now the conditional variance of  $y_t$  is determined as:

$$Var(y_t \mid \mu, \boldsymbol{\psi}, \mathbf{h}) = e^{h_t} + \psi_1^2 e^{h_{t-q}}.$$

So, we have that the conditional variance of  $y_t$  has two time-varying sources: the moving average of the most recent  $q + 1$  variances and the stationary AR(1) process followed by the log-volatilities, this last doing that the autocovariances of  $y_t$  be time-varying too. Unlike the AR(1)  $y_t$  is not serially independent and their autovariances are given by:

$$Cov(y_t, y_{t-j} \mid \mu, \boldsymbol{\psi}, \mathbf{h}) = \sum_{i=0}^{q-j} \psi_{i+j} \psi_i e^{h_{t-1}}$$

$$j = 1, \dots, q, j > q = 0, \psi_0 = 1$$

Assuming a multivariate normal prior for  $\boldsymbol{\psi}$  and independence of all the prior distributions  $\mu \sim N(\mu_0, V_\mu)$ ,  $\mu_h \sim N(\mu_{h0}, V_{\mu_h})$ ,  $\phi_h \sim N(\phi_{h0}, V_{\phi_h})$  1 ( $|\phi_h| < 1$ ) and  $\sigma_h^2 \sim IG(V_{\phi_h}, S_h)$ , we have  $p(\mu, \boldsymbol{\psi}, \mu_h, \phi_h, \sigma_h^2) = p(\mu) p(\boldsymbol{\psi}) p(\mu_h) p(\phi_h) p(\sigma_h^2)$ . Taking advantage of the band structure of the covariance matrix of  $\mathbf{y}$ , Chan and Hsiao (2014) show how standard MCMC simulation techniques can be applied for fitting this moving average stochastic volatility (MASV) model, similarly to the simpler SV model.

Another extension shown in Chan & Hsiao (2014) can be helpful to deal with the heavy-tailed distributions, a typical fact of financial returns series as consequence of extreme values. The relevant model can be specified as:

$$y_t = e^{\frac{1}{2}h_t} \lambda_t^{\frac{1}{2}h_t} \varepsilon_t \quad (11)$$

$$h_t = \mu_h + \phi_h (h_{t-1} - \mu_h) + \zeta_t \quad (12)$$

$\varepsilon_h \sim N(0, e^{h_t})$  and  $\zeta_t \sim N(0, \sigma_h^2)$ , and  $\lambda_t$  independent of each other. As before, the stationarity assumption,  $|\phi_h| < 1$ , and the stationary distribution to initialize the states are also kept. After discussed alternative distributions, Chan & Hsiao argue that if

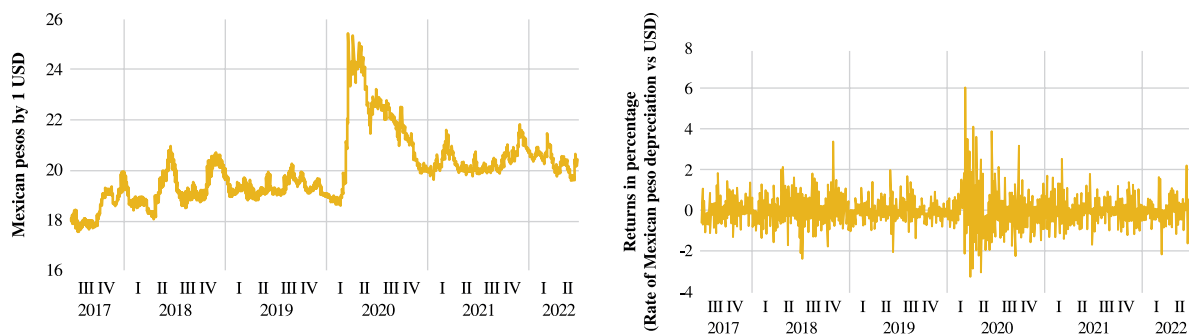
$$(\lambda_t | \nu) \sim IG(\nu/2, \nu/2), \quad (13)$$

then  $\tilde{\varepsilon}_t = \lambda_t^{-1/2} \varepsilon_t$  has a standard Student- $t$  distribution with  $\nu$  degrees of freedom. The model specification is completed with the independent prior distributions  $\mu \sim N(\mu_0, V_\mu)$ ,  $\nu \sim U(0, \bar{\nu})$ ,  $\mu_h \sim N(\mu_{h0}, V_{\mu h})$ ,  $\phi_h \sim N(\phi_{h0}, V_{\phi h})$  1 ( $|\phi_h| < 1$ ) and  $\sigma_h^2 \sim IG(v_h, S_h)$ . To take in account persistence, a MA(q) process can be included, arising a moving average stochastic volatility model with Student- $t$  errors (MASVt), whose estimation is carried similar to the SV model but with minor adjustments.

### III MXN PESO/US DOLLAR VOLATILITY ANALYSIS

Daily US dollar quotations, priced in Mexican pesos (MXN), were gathered from Banxico webpage ([www.banxico.org.mx](http://www.banxico.org.mx)) spanning from June 21<sup>st</sup>, 2017, to 21<sup>st</sup>, 2022, summing up a sample of 1,305 observations. As Figure 1 shows, the lowest US dollar rates were observed into the third quarter of 2017 and the highest occurred during the period of the Mexican economy lockdown associated with the COVID-19 pandemics emergence. The higher returns fluctuations of all the period occurred during the first three quarters of the same year, obviously also associated with the effects of the economic crisis induced by the pandemics. After the notorious rise of 2020, there was a change in the price level, which has remained around an average of 20.26; even consistently below that the level of 20 Mexican pesos for one US dollar in most of 2021 and during a good part of the period going until June 21<sup>st</sup>, 2022. It is worth no note that the post-pandemics average is slightly lower than the quotes from early December 2018 when the exchange rate reached 20.6 Mexican pesos for one US dollar. A general view of the returns over the whole period shows that 2020 is the period with the higher fluctuations and, after it, the fluctuation band has become narrower even than during the pre-pandemic period.

Figure 1. Mexican peso/US dollar exchange rate and returns (%): 6/21/2017-6/21/2022



Source: own elaboration based in **Banxico** webpage ([www.banxico.org.mx](http://www.banxico.org.mx)) and in our estimations.



**Table 1. Posterior means & standard deviations and relevant percentiles: daily MXN/US dollar exchange rate volatility (6/21/2017-6/21/2022)**

		Posterior		Percentile			
		Mean	Std. Dev.	1%	5%	95%	99%
SV	$\mu$	-0.0262	0.0171	-0.0659	-0.0542	0.0018	0.0142
	$\mu_h$	0.8040	0.2046	-1.2957	-1.1238	-0.4740	-0.2603
	$\phi_h$	0.9700	0.0100	0.9430	0.9521	0.9849	0.9899
	$\sigma_h^2$	0.0371	0.0101	0.0202	0.0235	0.0564	0.0657
MASV	$\mu$	-0.0261	0.0171	-0.0653	-0.0544	0.0017	0.0141
	$\mu_h$	-0.8051	0.2144	-1.3026	-1.1294	-0.4713	-0.2583
	$\phi_h$	0.9709	0.0097	0.9441	0.9540	0.9855	0.9908
	$\sigma_h^2$	0.0359	0.0094	0.0193	0.0230	0.0535	0.0641
	$\psi$	-0.0048	0.0289	-0.0718	-0.0530	0.0432	0.0627
MASV <sub>t</sub>	$\mu$	-0.0277	0.0172	-0.0681	-0.0556	0.0007	0.0124
	$\mu_h$	-0.8580	0.2213	-1.3938	-1.1930	-0.5161	-0.2921
	$\phi_h$	0.9726	0.0095	0.9468	0.9556	0.9868	0.9922
	$\sigma_h^2$	0.0329	0.0092	0.0168	0.0206	0.0492	0.0609
	$\psi$	-0.0050	0.0287	-0.0708	-0.0523	0.0418	0.0610
	$\nu$	33.9993	9.5732	12.3985	17.8672	48.3876	49.6399

SV, MASV & MASV<sub>t</sub> denote, respectively, vanilla, moving average & moving average with t-error stochastic volatility models. The analysis for each model is carried out based on 20,000 sampling draws after initial 1,000 burn-in draws.

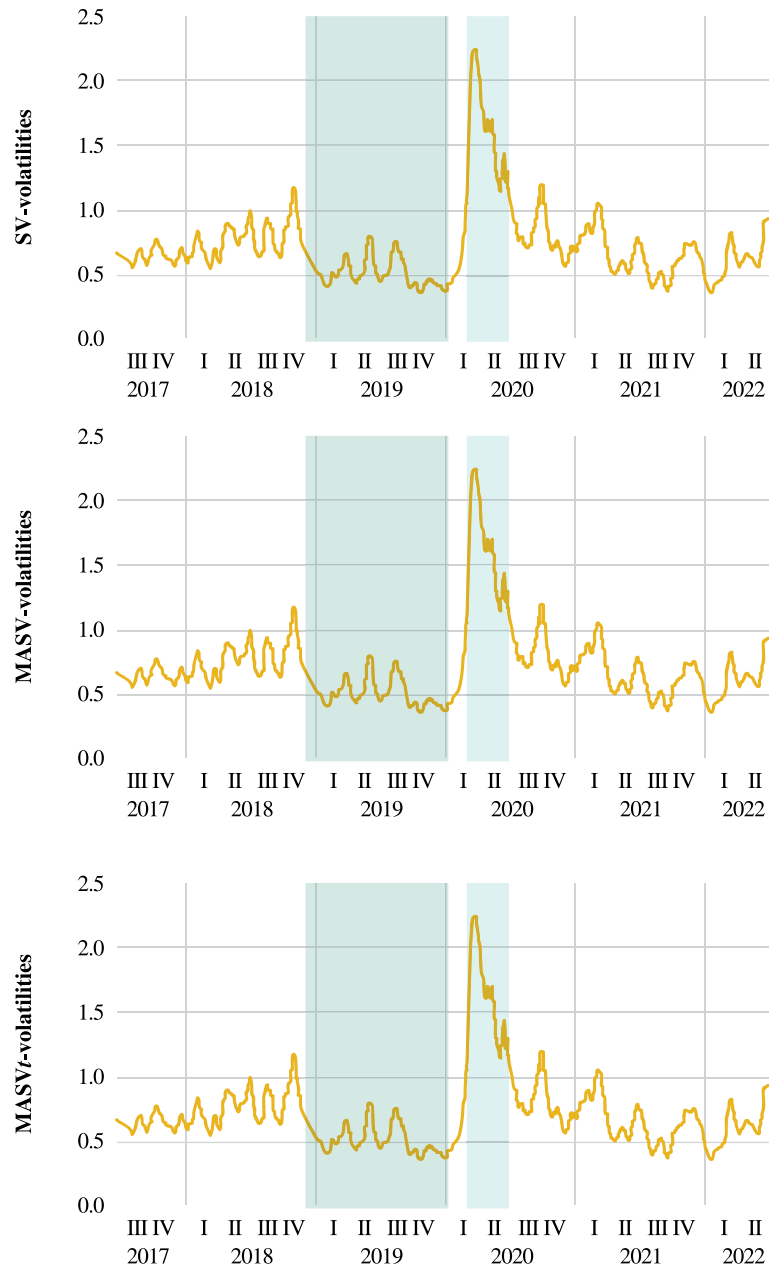
Source: own elaboration based in our estimations.

To adjust the three version of the stochastic volatility model, i.e. plain (SV), moving average (MASV) and moving average with Student-t errors (MASV<sub>t</sub>), the next values were set:  $\mu_0 = 0$ ,  $V_\mu = 5$ ,  $\mu_{h0} = 0$ ,  $V_{\mu h} = 5$ ,  $\phi_{h0} = 0$ ,  $V_{\phi h} = 1$ ,  $\nu_h = 10$  and  $S_h = 0.19$ . Considering a MA(1) process  $u_t = \varepsilon_t + \psi_1 \varepsilon_{t-1}$ , for  $\psi$  of the MASV model a truncated normal prior distribution  $N(\psi_0, V_\psi) 1(|\psi| < 1)$  was assumed with  $\psi_0 = 0$  and  $V_\psi = 1$ . For the prior distribution of  $\nu$  in the MASV<sub>t</sub> model,  $\bar{\nu} = 50$  was set. For each model 20,000 draws from the posterior distribution were obtained after a burn-in period of 1,000.

Table 1 shows the posterior means and standard deviations of the estimated parameters, so as their relevant quantiles. The Gaussian models (SV and MASV) provides similar estimations (-2.62% vs -2.61%) for the mean of  $\mu$ , the mean daily return for the period, while the model with Studentized errors gives a relative higher estimation (-2.77%) with only a slightly higher uncertainty (1.72%). Nevertheless, all the models shows that the relevant intervals of credibility (90 and 95 percent) do not exclude the zero value, so it is not possible to claim that the mean return is not likely to be around zero from June 21<sup>st</sup>, 2017, to June 21<sup>st</sup>, 2020.

At the other hand, the three models suggest that  $\mu_h$ , the estimated constant level of the log-volatility, does not cross the zero value, even though the three models differ about which is the mean of the level, with the MASV<sub>t</sub> suggesting a higher posterior mean. Also the three models show similar posterior means for the parameter  $\phi_h$  which captures the AR process of the log-volatilities, providing values around the expected with basis on the previous research suggesting high persistent volatility.

Figure 2. Mexican peso/US dollar posterior means of the time-varying volatilities,  $e^{h_t}$ :  
6/21/2017-6/21/2022



Source: own elaboration based in our estimations.

It is worth to note that the two versions of the stochastic volatility model including moving average process, suggest similar posterior means and standard deviation for the relevant parameter  $\psi$ , but the respective credibility intervals includes zero value, suggesting that this parameter fluctuates around zero into the period of this analysis. So, it can be regarded as a non-rejection of the hypothesis test  $\psi = 0$ , that is, this test result suggests that, in comparison with the two moving average alternatives estimated, the estimates provided by the SV model specification can be sufficient to capture the volatility dynamics of the Mexican peso/USD exchange rate.

Figure 2 shows the posterior means of the time-varying volatilities (standard deviations) of the three *estimated* stochastic volatility models. One can immediately notice the high degree of similarity between the estimated volatilities, to the point that they are indistinguishable from each other; so the following comments are valid for all three series. At first glance stand out the higher levels of the volatilities during the period associated to the uncertainty produced by the outbreak of the pandemics during 2020. It can also be seen that during 2019 the level of volatility was the lowest of the entire period, as shown by the dark shaded area, even being below 0.5% for several consecutive days at the beginning of that year and during the final two months, and in the other days of 2019 the level was only a little bit higher, near the mean 2017 level as much, but always in a lower level than the volatilities observed in 2018.

In the last days of October, 2018 is observed the higher level of the exchange rate volatility prior to the formal day of change of administration, December 1<sup>st</sup> 2018. Such high could be related to the announcement of the results of the public consultation on the continuation of the new Mexico City airport in Texcoco, according to which its location should be changed. Nevertheless, after this high, the volatility level dropped constantly reaching a new level for all 2019, notably lower than the previous ones. The volatility came back to former levels until February 2020, as could be expected given the pessimistic views in front the speed of the COVID-19 pandemic spread and the war of prices in the oil market. The increase in volatility went beyond previous levels, reaching the maximum of the entire horizon covered by this study in the last week of March of the same year, after which a decline is observed with a strongly sustained trend.

As also shown in Figure 2 (light shaded area), after the outburst of volatilities produced by the pandemic's crisis, their levels have become very similar to that observed before the critical period. The volatilities corresponding for the 2021 first quarter are very similar to those observed during 2018, but with a descending trend that push them to the levels of 2019. Although this second lower level of volatility is sustained until the 2022 first quarter, an increase is observed at the end of the sample, obviously, it can be associated to the burst of the war in Ucrania.

## CONCLUDING REMARKS

This paper shows the results of the volatility analysis of the Mexican peso/US dollar exchange rate that was carried out using three stochastic volatility models estimated by Markov Chain Monte Carlo (MCMC) methods. The period of the analyzed sample (6/21/2017 to 6/21/2022) is interesting because it facilitates the study of volatility of this type in a context of an important political change in the Mexican government, as well as the emergence of COVID-19 pandemics that, although not being an event of economic nor political nature, provoked one of the worst economic crises known up today, with effects throughout the world and, at last but not the least, the outbreak of a war that contributed to worsen the pessimistic prospects about the world's economic performance.

From a general view, it is convenient to highlight that the estimated volatilities through the three estimated models show the impact of the mandatory closure of economic activities to face the COVID-19 pandemic, an unexpected event that clearly increased uncertainty in the Mexican foreign exchange market. It is also convenient to emphasize that after the general economic activity was restarted, although gradually and with different rhythms in the federative entities and the different sectors, the volatility of the exchange rate Mexican peso/US dollar dropped returning to the levels observed in the previous year to the irruption of the pandemic, levels that according to the estimates were of the lowest in the period.

Absolutely, the low uncertainty levels around the exchange rate regards the USD observed in our analysis can be explained by the US economic expected performance, as some specialists in economic issues have claimed.

But observing the local economic performance, in order to gain a better understanding of the exchange rate dynamics, it is also necessary to understand the role played by the real fundamentals of the Mexican economy, so as the role played by the policies focused in the social development.

In a society with as much socioeconomic inequality as Mexico's, public spending policy seems to have contributed both in the alleviation of the economic problem produced by the pandemic, and in the incorporation of large layers of society to levels of consumption and satisfaction of basic needs that they lacked. It seems obvious that the higher level of consumption induces more investment, triggering higher levels of dynamics in the overall economic activity. So, to evaluate the performance of the AMLO presidential term, the research question could be about the impact of such economic policies on the wellbeing of the Mexican people. At this regard, the results of our inquiry on the exchange rate uncertainty could be explained, in last instance, as a reflection of that policies.

On the other hand, from the methodological perspective adopted by this analysis, it can be seen that in the period covered by the study the models provide very similar results. However, it is important to consider that the credibility intervals that were estimated by the sampling algorithms used suggest that the simplest stochastic volatility model is sufficient to capture the level of relative uncertainty in the exchange rate during the period studied. The dramatic change observed in the exchange parity as a consequence of the outbreak of the aforementioned pandemic did not have such important effects as to alter the accumulated density in the tails of the probability distribution, moving it away from a normal distribution, nor did it produce a higher degree in the persistence of past volatility. So, this result could be suggesting that, except for the highest period of the crisis, there is a high degree of stability in the exchange parity between both currencies.

Based on the results of this analysis, a research agenda related to exchange parity between the Mexican peso and the US dollar can be raised. In principle, it could be suggested that the analysis of the high increase in volatility observed during the emergence of pandemic go deeper in order to understand how much of this volatility corresponded to the real fundamentals of the Mexican economy and how much was the result of an overshooting in which speculative activities could even have played an important role besides the deterioration of expectations around the consequences in the economic performance. Another important issue within that research agenda could be the determination of the effects that the sanitary and economic measures had with which the violent appearance of the pandemic faced; as well as economic policy measures may have contributed to maintaining that stability of the Mexican weight even despite the economic crisis derived from the pandemic.

Finally, as an important byproduct, it is also important to consider that the results provided by the three models estimated in the analytical framework of this study, also suggests the relevance of the use of stochastic volatility models, given the remarkable advance in methods and computing power to estimate even versions with variations that can be very useful to study more complex contexts. Such contexts could be as that faced in other more risky financial markets or, even, important commodities markets subject to higher degrees of stress by the economic expectations or speculative.

## REFERENCES

- Agosin, M. and Díaz, J. D. (2023). Explaining the volatility of the real exchange rate in emerging markets. *International Review of Economics and Finance*, 87, 110-123. <https://doi.org/10.1016/j.iref.2023.04.024>
- Avilés Ochoa, E. and Flores Sosa M.M. (2021). Comparison of the GARCH and stochastic models: An application to the Mexican peso-us dollar exchange rate. *Contaduría y Administración*, 66(2), 1-14. <http://dx.doi.org/10.22201/fca.24488410e.2021.2642>

- Beckmann, J. and Czudaj, R. L. (2022). Exchange rate expectation, abnormal returns, and the COVID-19 pandemic. *Journal of Economic Behavior & Organization*, 196, 1-25. <https://doi.org/10.1016/j.jebo.2022.02.002>.
- Black, F. and Scholes, M. (1973) The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81(3), 637-654. <https://www.jstor.org/stable/1831029>
- Chan, J.C.C. and Hsiao, C.Y.L. (2014). Estimation of Stochastic Volatility Models with Heavy Tails and Serial Dependence. In: I. Jeliaskov and X.S. Yang (Eds.), *Bayesian Inference in the Social Sciences*, 159-180, John Wiley & Sons, New York.
- Chan, J.C.C. and Jeliaskov, I. (2009). Efficient Simulation and Integrated Likelihood Estimation in State Space Models, *International Journal of Mathematical Modelling and Numerical Optimization*, 101-120. <https://doi.org/10.1504/IJMMNO.2009.03009>
- Chuanjian, Li.; Su, Z-W.; Yaqoob, T. and Sajid, Y. (2021). COVID-19 and currency market: a comparative analysis of exchange rate movement in China and USA during pandemic, *Economic Research-Ekonomska Istraživanja*, <https://doi.org/10.1080/1331677X.2021.1959368>
- Corsetti, G. & Marin, E. A., 2020. A Century of Arbitrage and Disaster Risk Pricing in the Foreign Exchange Market. *Cambridge Working Papers in Economics*, Faculty of Economics, University of Cambridge.
- Devpura, N. (2021). Effect of COVID-19 on the relationship between Euro/US dollar exchange rate and oil price. *MethodsX*, 8, 101262. <https://doi.org/10.1016/j.mex.2021.101262>
- Engle, R.F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, 50(4), pp. 987-1007. <https://doi.org/10.2307/1912773>
- Erer, D. (2023). The Impact of News Related COVID-19 on Exchange Rate Volatility: A New Evidence From Generalized Autoregressive Score Model. *EKOIST Journal of Econometrics and Statistics*, 0(38), 105-126. <https://doi.org/10.26650/ekoist.2023.38.1179575>
- Giofré, Maela and Sokolenko O. (2023). Cross-border investment and the decline of exchange rate volatility: implications for Euro area bilateral investments. *Review of World Economics*, 159, 595–627. <https://doi.org/10.1007/s10290-022-00477-y>
- Hamilton, J.D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, 57(2), 357-384.
- Harvey, A., Ruiz, E. & Shephard, N. (1994). Multivariate Stochastic Variance Models. *The Review of Economic Studies*, 61(2), 247-264. <http://www.jstor.org/stable/2297980>.
- Kim S., Shepherd N. and Chib S. (1998). Stochastic Volatility: Likelihood Inference and Comparison with ARCH Models. *Review of Economic Studies*, 65(3), 361–393. <https://doi.org/10.1111/1467-937X.00050>
- Lintner, John (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *The Review of Economics and Statistics*, 47(1), 13-37. <https://doi.org/10.2307/1924119>
- López-Herrera, F.; Rodríguez Benavides, D. and Ortiz Arango, F. (2011). The stochastic volatility of the peso-dollar exchange rate: the floating regime in Mexico. *Investigación Económica*, 70(276), 19-50.
- Markowitz, H. M. (1952). Portfolio Selection. *Journal of Finance*, 7(1), 77-91. <https://doi.org/10.2307/2975974>
- Markowitz, H. M., (1957). A Simplex Method for the Portfolio Selection Problem. *Cowles Foundation Discussion Papers*, 244. <https://elischolar.library.yale.edu/cowles-discussion-paper-series/244>.
- Markowitz, H. M. (1959), *Portfolio Selection: Efficient Diversification of Investments*, Cowles Foundation for Research in Economics at Yale University, Monograph No. 16. New York, USA.
- Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica*, 34, 768-783.
- Narayan, P. K. (2020). Has COVID-19 Changed Exchange Rate Resistance to Shocks? *Asian Economics Letters*, 1(1). <https://doi.org/10.46557/001c.17389>

- Narayan, P.K.; Gong, Q. and Ahmed, H.J.A. (2020). Is there a pattern in how COVID-19 has affected Australia's Stock Returns? *Applied Economic Letters*, 29(3), 179-182. <https://doi.org/10.1080/13504851.2020.1861190>.
- Pasiouras, A., and Daglis, T. (2020). The dollar exchange rates in the COVID-19 era: Evidence from 5 currencies. *European Research Studies Journal*, 23(Special 2), 352–361. <https://doi.org/10.35808/ersj/1828>
- Sharpe, William F. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk (1964). *Journal of Finance*, 19(3), 425-442. <https://doi.org/10.1111/j.1540-6261.1964.tb02865.x>
- So, M. K.P., Chu, Amanda M.Y., Lo, C.Y. and Ip, Ch. Y. (2021). Volatility and dynamic dependence modeling: Review, applications, and financial risk management. *WIREs Computational Statistics*, Early View, e1567, 1-21. <https://doi.org/10.1002/wics.1567>
- Taylor, S. J. (1982). Financial returns modelled by the product of two stochastic processes-A study of the daily sugar prices 1961–1975. In *Time Series Analysis: Theory and Practice 1*. Edited by Anderson O. D. Amsterdam: North Holland, pp. 203–26.
- Taylor, S. J. (2005). *Asset price dynamics, volatility, and prediction*, Princeton University Press, Princeton, N.J.
- Treynor, J. L. (1961). Market value, time, and risk. <http://dx.doi.org/10.2139/ssrn.2600356>.
- Treynor, J. L. (1962). Toward a *Theory of Market Value of Risky Assets*. <http://dx.doi.org/10.2139/ssrn.628187>
- Villarreal-Samaniego, D. (2021). The dynamics of oil prices, COVID-19, and exchange rates in five emerging economies in the atypical first quarter of 2020. *Estudios Gerenciales*, 37(158), 17-27. <https://doi.org/10.18046/j.estger.2021.158.4042>